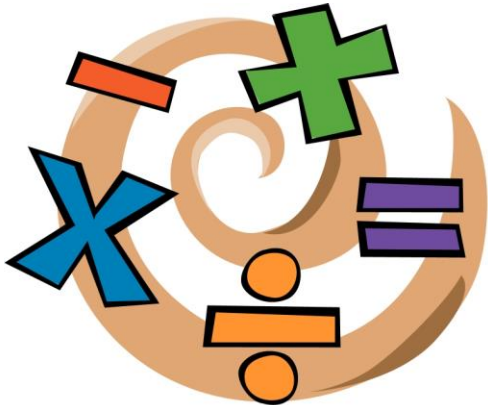
Four Lanes Community Junior School

Inspire and Achieve Together

Calculation Progression Policy 2020



Aims and objectives

This policy aims to standardise which strategies will be taught and how children will record their calculations.
This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

The division of skills and methods within this calculation policy are based on the requirements of the National Curriculum for the teaching and learning of mathematics and takes into account the progression through our school. Although this policy is set out in year groups, it is important that we consider the prior attainment of our children and move on only when they are ready to take on the next challenge. Mathematical understanding is developed through use of representations that are first of all **concrete** (e.g. base ten, apparatus), then **pictorial** (e.g. array, place value counters) to then facilitate **abstract** working (e.g. columnar addition, long multiplication). Children should not be expected to move onto the next stage if they are not ready or if they are not confident.

The National Curriculum for Mathematics (2014) aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

At Four Lanes Juniors we believe that any calculation needs to have a context so that its purpose can be understood. We provide our children with many opportunities to use and apply their mathematical skills in different situations and to develop their reasoning skills. Our children are taught a variety of methods, both mental and written, so that they develop the skills required to select an efficient method which is determined by the calculation. Children are encouraged to consider if a mental calculation would be appropriate before using written methods.

Children should be encouraged to approximate their answers before calculating and to use a suitable strategy to check for accuracy. Children are taught to use methods which are accurate, reliable and efficient.

When faced with a calculation problem, encourage your child to ask…

- Can I do this in my head?
- Could I do this in my head using drawings or jottings to help me?
- Do I need to use a written method?

Also help your child to estimate and then check the answer. Encourage them to ask…

- Is the answer sensible?

When are children ready for written calculations?

These lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

Addition and subtraction	Multiplication and Division
• Do they know addition and subtraction facts to 20? • Do they understand place value and can they partition numbers? • Can they add three single digit numbers mentally? • Can they add and subtract any pair of two-digit numbers mentally? • Can they explain their mental strategies orally and record them using informal jottings? • Can they confidently count on and back? • Can they confidently bridge through 10?	• Do they know the 2, 3, 4, 5 and 10 times tables? • Do they know the result of multiply by 0 and 1? • Do they understand 0 as a place holder? • Can they multiply two- and three-digit numbers by 10 and 100 and explain the effect? • Can they double and halve two-digit numbers mentally? • Can they use multiplication facts that they already know to derive mentally other multiplication facts that they do not know? • Can they explain their mental strategies orally and record them using informal jottings?

<u>Year Group Calculation Method Overview</u>

(Teachers are to use their own judgements as to whether individual children are ready to use the methods listed in their year group or if they need to consolidate previous learning before moving on to these)

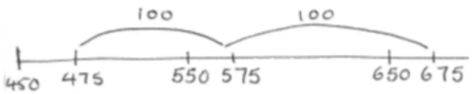
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Exchanging to make 10.	Adding three single digits. Column method – no exchanging.	Column method - exchanging (up to 3 digits)	Column method - exchanging (up to 4 digits)	Column method – exchanging (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method - exchanging. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no exchanging	Column method with exchanging (up to 3 digits)	Column method with exchanging (up to 4 digits)	Column method with exchanging (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with exchanging. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Multiplication within a place value grid	Column multiplication (2 and 3 digits multiplied by 1 digit)	Column multiplication (up to 4-digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1-digit - concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1-digit - concrete and pictorial)	Short division (up to 4 digits by a 1-digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2-digit number- Interpret remainders as whole numbers, fractions or round)

MENTAL STRATEGIES FOR ADDITION

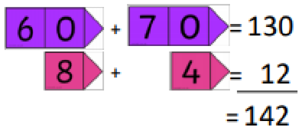
Informal jottings (for example number lines) and concrete apparatus will be used to develop an understanding of conceptual relationships

Year 3 ADDITION

Using Place value



Count in hundreds e.g. knowing 475+200 as 475, 575, 675
Add multiples of 10, 100 and £1 e.g. 746+200 or 746+40
Use partitioning:
68 + 74 as 60 + 70 and 8 + 4 , combining the two totals
(Use arrow cards to help this method – it reinforces place value knowledge)



Add three-digit numbers by adding h, t and o e.g. 452 + 321 as 452 + 300 + 20 + 1

Counting on

Add three-digit and two-digit numbers by adding h, t and o
e.g. 125 + 34 as 100 + 20 + 30 + 5 + 4
Add near multiples of 10 and 100
e.g. 67 + 39 as (67 + 40) – 1

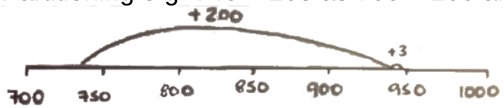
Using number facts

Number bonds to 100 e.g. 36 + 64, 83 + 17 etc.
Adding to next ten and next hundred e.g. 176 + 4 = 180, 435 + 65 = 500

Year 4 ADDITION

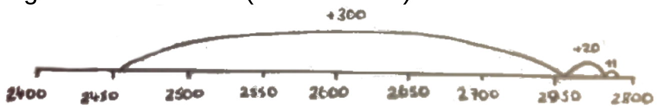
Using place value

Count in thousands e.g. knowing 1475+2000 as 1475, 2475, 3475
Add multiples of 10, 100 and £1 e.g. 746+200 or 746+40
Partitioning e.g. 746 + 203 as 700 + 200 and 46 + 3 or 746 + 200 + 3



Counting on

Add a three-digit and four-digit numbers by adding th, h, t and o
e.g. 2452 + 321 as 2452 + 300 + 20 + 1
Add near multiples of 10, 100 and 1000
e.g. 3462 + 2999 as (3462 + 3000) – 1



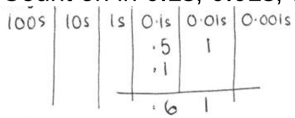
Using number facts

Number bonds to 100 and to the next multiple of 100 e.g. 1353 + 47
Adjustment when adding 9 and 11
e.g. 27 + 9 as (27 + 10) – 1 or 36 + 11 as (36 + 10) + 1

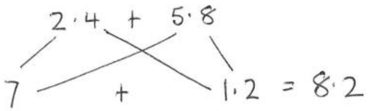
Year 5 ADDITION

Using place value

Count on in 0.1s, 0.01s, e.g. knowing what 0.1 more than 0.51 is



Partitioning e.g. 2.4 +5.8



Counting on

Count on from the greatest number e.g. 6834 + 3005 as 9834 + 5
Add near multiples e.g. 82,472 + 30,004 as (82,472 + 30,000) + 4
Add two decimal numbers by adding the o then ths then hths e.g. 5.72 + 3.05 as 5.72 + 3 + 0.5

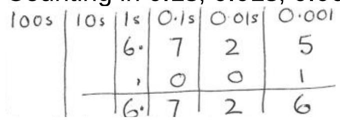
Using number facts

Know number bonds to 1 and to the next whole number e.g. 0.4 + 0.6 or 5.7 +0.3
Add to the next 10 from a decimal number e.g. 7.8 + 2.2 = 10

Year 6 ADDITION

Using place value

Counting in 0.1s, 0.01s, 0.001s e.g. knowing what 0.001 more than 6.725 is



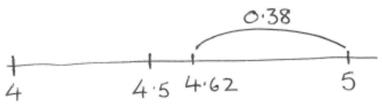
Partitioning e.g. 9.54 + 3.25 as 9 + 3 and 0.5 + 0.2 and 0.04 + 0.05 to get 12.79

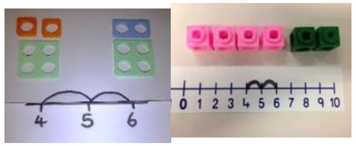
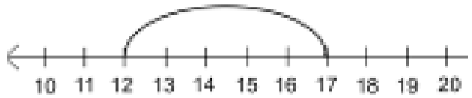
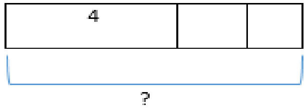
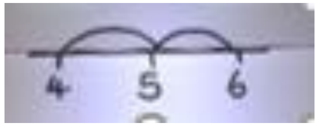
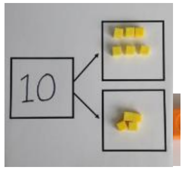
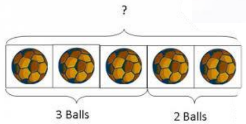
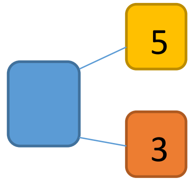
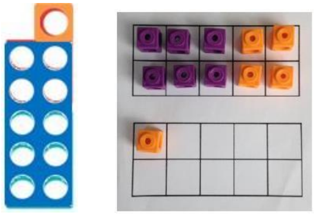
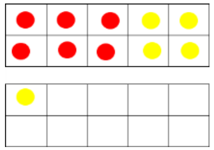
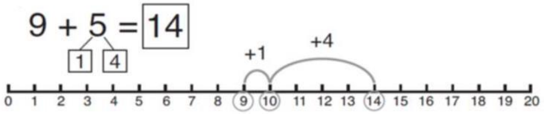
Counting on

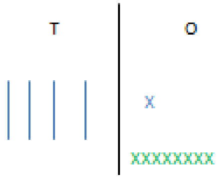
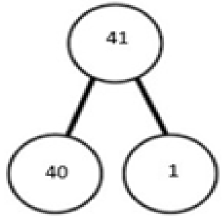
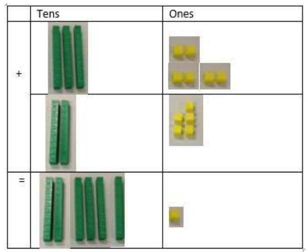
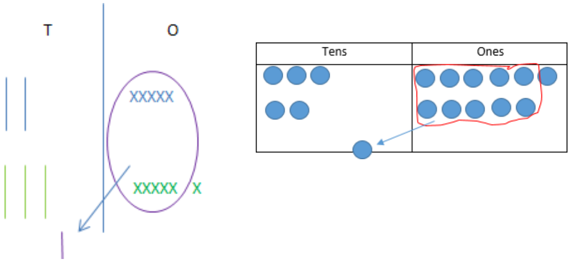
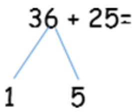
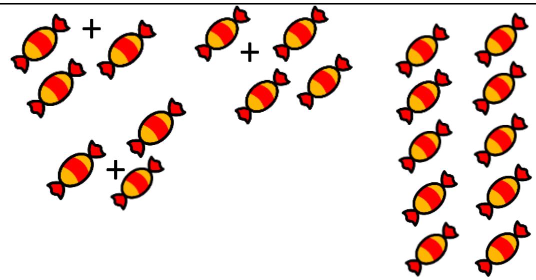
Add two decimal numbers by adding the o then ths then hths or tths
e.g. 6.31 + 3.46 as 6.31 + 3 + 0.4 + 0.06
Add near multiples to 1 and 10 e.g. 6.75 + 9.95 as (6.75 + 10) – 0.05 or 6.35 + 0.99 as (6.35 + 1) – 0.01

Using number facts

Number bonds to 1 and the next multiple of 1, e.g. 0.63+0.37 or 2.355+0.645
Add to next ten, e.g. 4.62+0.38

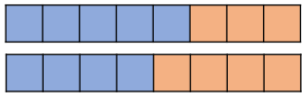


End of Year Expectations (National Curriculum Statements)	**Teaching and Learning Strategy**	**Concrete**	**Pictorial**	**Abstract**
Key Stage 1 Addition: • read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs • represent and use number bonds within 20 • add one-digit and two-digit numbers to 20, including zero • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 9 = ? + 6 • solve problems with addition: ➢ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ➢ applying their increasing knowledge of mental and written methods • recall and use addition facts to 20 fluently, and derive and use related facts up to 100 • add numbers using concrete objects, pictorial	Counting on	Counting on using number lines by using cubes or Numicon.   Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	**12 + 5 = 17**  Start at the larger number on the pre-made number line and count on in ones or in one jump to find the answer. A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 =  Place the larger number in your head and count on the smaller number to find your answer. **5 + 12 = 17**
	Combining two parts to make a whole: part-whole model	   Use cubes and other resources/objects to add two numbers together as a group or in a bar.	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
	Regrouping to make 10.	 Regrouping to make 10 by using ten frames and counters/cubes or using Numicon: 	Children to draw the ten frame and counters/cubes   Use pictures or a number line. Regroup or partition the smaller number to make 10.	Children to develop an understanding of equality e.g. 6 + □= 11 and 6 + 5 = 5 + □ and 6 + 5 = □+ 4

representations, and mentally, including: ➢ a two-digit number and ones ➢ a two-digit number and tens ➢ two two-digit numbers ➢ adding three one-digit numbers • show that addition of two numbers can be done in any order (commutative) • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	Adding TO + O using base 10.	Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	41 + 8: 1 + 8 = 9 40 + 9 = 49 
	TO + TO using base 10.	**36 + 25** 		Looking for ways to make 10: 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61  Formal method: 36 +25 61 1
	Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.

Examples of Key Stage 1 varied fluency

How can we use the following representation to prove that 5 + 3 = 4 + 4?



Can you use inverse operations to check 5 + 12 = 17?

How many possible inverse calculations are there?

Eva writes this calculation: 18 – 5 = 13
Which of the following could she use to check her work?

13 + 5	13 – 5
18 – 13	5 + 13

Complete the missing numbers.

5 + 3 = 6 + _____

5 + 3 = _____ + 6 = 7 + _____

_____ + 3 = _____ + 4 = 5 + 5

Fill in the circles with either <, > or =

6 + 4 ◯ 6 + 5

6 + 4 ◯ 3 + 6

Examples of Key Stage 1 reasoning and problem solving

How many different ways can you solve 19 + 11?

Explain your method to a partner.

Use concrete or pictorial resources to help explain your method.

Always, Sometimes, Never

odd + odd + odd = odd

Use one-digit numbers to test if this is true e.g.

3 + 5 + 7

Which numbers would you add together first in the following number sentences? Why would you add those first?

3 + 5 + 7 =

8 + 2 + 6 =

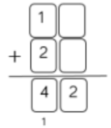
4 + 3 + 4 =

Is there always an easier order to add three one-digit numbers?

What digits could go in the boxes?

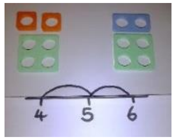
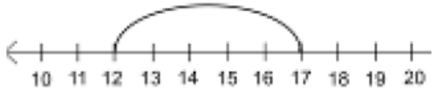
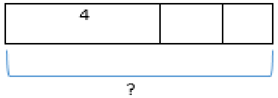
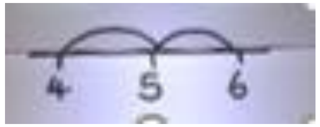
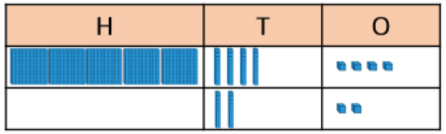
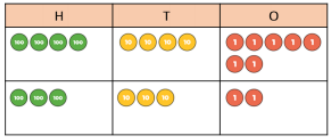
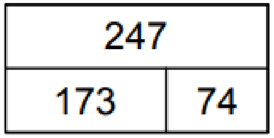
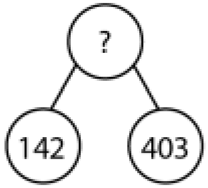
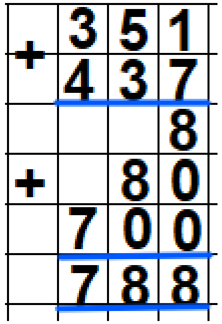
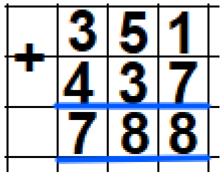
☐2 + ☐5 = 87

Find all the possible pairs of numbers that can complete the addition.



How do you know you have found all the pairs?

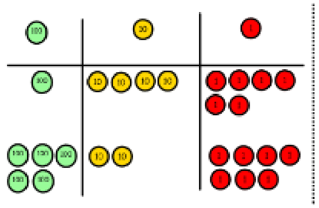
What is the same about all the pairs of numbers?

End of Year Expectations (National Curriculum Statements)	**Teaching and Learning Strategy**	**Concrete**	**Pictorial**	**Abstract**
Year 3 Addition: • add numbers mentally, including: ➢ a three-digit number and ones ➢ a three-digit number and tens ➢ a three-digit number and hundreds • add numbers with up to three digits, using formal written methods of columnar addition • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	Counting on	Counting on using number lines by using cubes or Numicon e.g. 4 + 2 =  Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer e.g. 10 + 2 = 	**12 + 5 = 17**  Start at the larger number on the pre-made number line and count on in ones or in one jump to find the answer. A bar model which encourages the children to count on e.g. 4 + 2 = 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What is the total of 4 and 2? 4 + 2 =  Place the larger number in your head and count on the smaller number to find your answer.
	Column method - no exchanging HTO + O HTO + TO HTO + HTO	 544 + 22 = 566 Add together/count ones then tens then hundreds.  ___ + ___ = ___ 447 + 332 = 779	173 + 74 = 247  Use bar models and part-whole models to help children see that the total is equal to the two numbers being added.  142 + 403 = ? (525)	Expanded column method: Move to adding the least significant digit first. Secure place value and understanding of how numbers can be partitioned in different ways is vital. HTO + HTO 351 + 437 300 + 50 + 1 400 + 30 + 7 700 + 80 + 8 = 788 Once children are secure in understanding the place value of the numbers they are adding move them onto using more compact column methods. 

Column method - exchanging

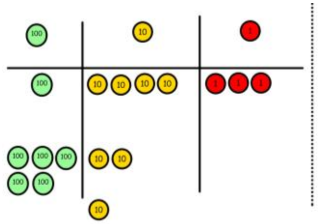
HTO + O
HTO + TO
HTO + HTO

Make both numbers on a place value grid.



146
+ 527

Add up the ones and exchange 10 ones for one 10.

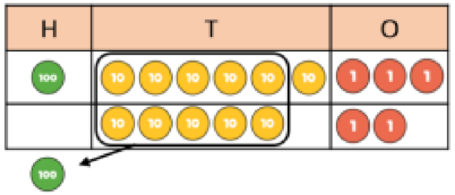


146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

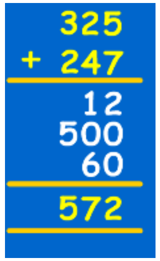
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

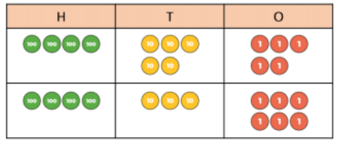
Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



163 + 52 = 215

Use expanded column addition with exchanges before moving onto the formal written method. This can be supported by concrete resources to aid transition to abstract methods.





	4	5	5
+	4	3	6

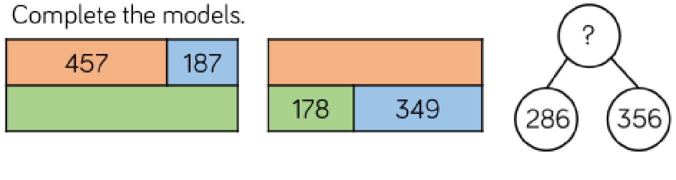
Examples of Year 3 varied fluency

Complete each box for 400 + 500

Draw It	Write It	Part-Whole	Number Sentence
	__ hundreds and __ hundreds is equal to __ hundreds		__ + __ = __

Miss Wilson has 237 marbles in a box.
She adds 8 more bags of 10 marbles.
How many marbles does she have now?
Write the calculation for this problem.

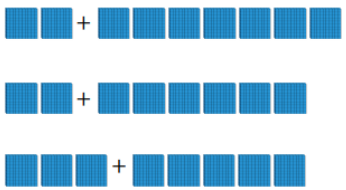
Complete the models.



Examples of Year 3 reasoning and problem solving

Which is the odd one out?

Explain why.

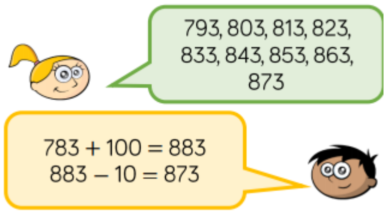


Always, Sometimes, Never

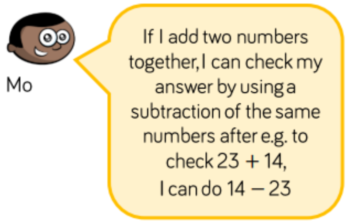
When 7 and 5 are added together in the ones column, the digit in the ones column of the answer will always be 2

What other digits would always give a 2 in the ones column? Prove it.

Eva and Amir are calculating 783 + 90



Whose method do you prefer? Explain why.



Do you agree? Explain why.

Complete the statements to make them correct.

487 + 368 ◯ 487 + 468

326 + 258 ◯ 325 + 259

391 + 600 = 401 + ___

Explain why you do not have to work out the answers to compare them.

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract

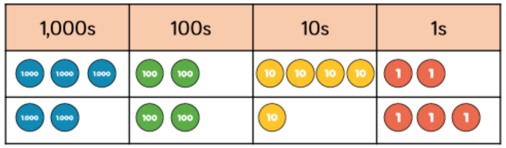
Year 4 Addition:

- add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

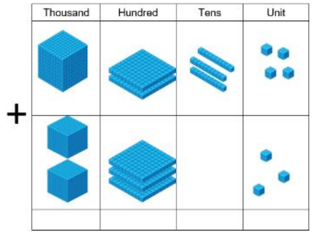
Column method - no exchanging

ThHTO + O
ThHTO + TO
ThHTO + HTO
ThHTO + ThHTO

Concrete: Use counters and a place value grid

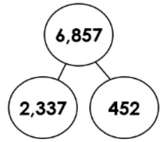


3242 + 2213 =5455



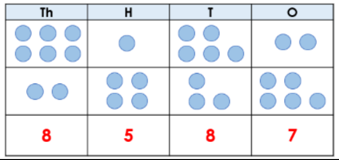
1234 + 2303 = 3537

Pictorial:



Use part-whole model to link to prior learning on addition. Make children aware that this will not be the most efficient method for addition of 4-digit numbers but is something we can use to represent a calculation.

Children can draw pictorial representations of columns and place value counters before moving onto the formal written method.



6152 + 2435 = 8587

Abstract: Encourage children to continue to use concrete resources alongside to support their working out whilst recording their work using the formal written method. Begin by showing the expanded column method and then move onto compact addition.

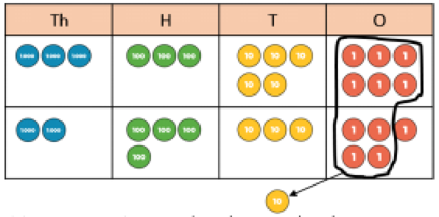
	7	1	5	3
+	1	7	0	4
				7
+			5	0
		8	0	0
	8	0	0	0
	8	8	5	7

	7	1	5	3
+	1	7	0	4
	8	8	5	7

Column method - exchanging

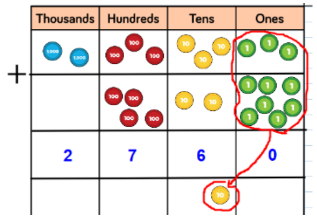
ThHTO + O
ThHTO + TO
ThHTO + HTO
ThHTO + ThHTO

Concrete: 3356 + 2435 = 5791



Regroup and exchange when columns exceed 9 counters

Pictorial: Draw representations of exchanging:
2333 + 427 = 2760



3 + 7 = 10 so you exchange 10 ones for 1 ten – shown by drawing the counters moving into the next column as 1 ten.

Abstract:

	4	1	4	6
+	3	5	3	7
			1	3
+			7	0
		6	0	0
	7	0	0	0
	7	6	8	3

	4	1	4	6
+	3	5	3	7
	7	6	8	3
			1	

Formal written method with exchanging, modelling how to show exchange in calculations. Use expanded columns to show what is happening with the exchange.

Examples of Year 4 varied fluency

Use <, > or = to make the statements correct.

3,456 + 789 ◯ 1,810 + 2,436

2,829 + 1,901 ◯ 2,312 + 2,418

Work out the missing numbers.

	Th	H	T	O
	4	___	6	___
+	2	5	___	1
	___	7	8	9

Match the calculations with a good estimate.

345 + 1,234	3,000 + 6,000
2,985 + 6,325	3,500 + 1,200
3,541 + 1,179	350 + 1,200
2,135 + 6,292	2,000 + 6,000

True or false? Use column addition to check the answer.

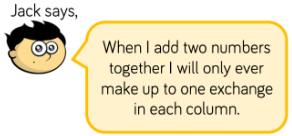
8,179	
4,065	4,114

Examples of Year 4 reasoning and problem solving

Find 3 different number sentences to complete this calculation.

	5	☐	5	0
+	☐	☐	☐	☐
	8	7	8	4

Jack says,



Do you agree?
Explain your reasoning.

The estimated answer to a calculation is 3,400
The numbers in the calculation were rounded to the nearest 100 to find an estimate.
What could the numbers be in the original calculation?

Use the digit cards to create addition calculations using two 4-digit numbers with three exchanges.

5 9 3 6

Find two possibilities.

Find and explain the mistakes.

	4	7	8	2
+	1	4	2	0
	6	1	0	2
	1	1		

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract

Year 5 Addition:

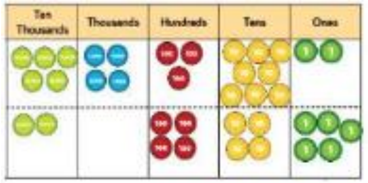
- add whole numbers with more than 4 digits, including using formal written methods
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- practise adding decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1

Teaching and Learning Strategy: Column method

TThThHTO + O
TThThHTO + TO
TThThHTO + HTO
TThThHTO + THTO
TThThHTO + TThThHTO

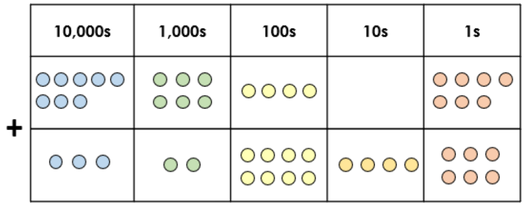
Concrete:

What is the total? 54382 + 20445 = 74827



Use dienes or counters within a place value grid to add whole numbers with more than 4 digits. Reinforce methods of exchanging learned in previous years.

Pictorial:



86407 + 32846 = 129253

Draw representations of counters to add whole numbers with more than 4-digits, with and without exchanging.

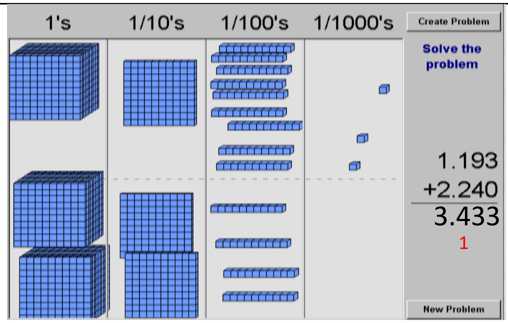
Abstract:

	2	3	3	1	6
		4	0	8	4
+	5	1	2	2	5
	7	8	6	2	5
			1		1

Formal column method for exchanging and non-exchanging calculations.

Teaching and Learning Strategy: Column method – addition of decimal numbers

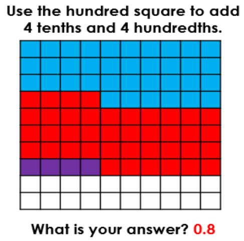
Concrete:



Use place value grids with counters or dienes. Reinforce use of 0 as a place holder when adding numbers with different amounts of decimal places.

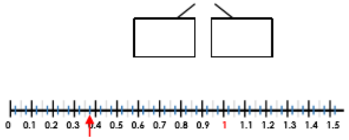
Pictorial:

Use hundred squares to colour and add decimals together – link to fractions knowledge.



Use partitioning to find a complement to 1 and then a number line to solve the calculation.

0.375 + 0.925



Abstract:

	0.	8	1	6
+	0.	1	2	5
	0.	9	4	1
			1	

	1	9.	0	1
+		3.	6	5
		0.	7	0
	2	3.	3	6
		1		

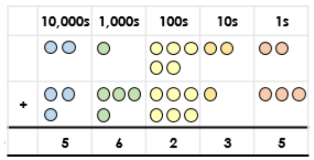
Use formal written methods to add decimals, including addition of decimals and whole numbers and decimals with different numbers of decimal places.

Examples of Year 5 varied fluency

Jack, Rosie and Eva are playing a computer game. Jack has 3,452 points, Rosie has 4,039 points and Eva has 10,989 points.

How many points do Jack and Rosie have altogether?
How many points do Rosie and Eva have altogether?
How many points do Jack and Eva have altogether?
How many points do Jack, Rosie and Eva have altogether?

True or false?



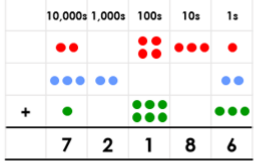
Which is best to estimate the total of 22,223 and 5,687?

22,300 + 5,700
22,200 + 5,700
22,200 + 5,600

I'm thinking of a number.
After I add 5,241 and subtract 352, my number is 9,485
What was my original number?

Examples of Year 5 reasoning and problem solving

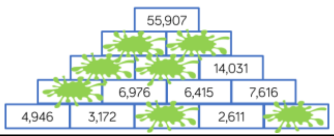
Add the missing place value counters to make this addition correct.



Work out the missing numbers.

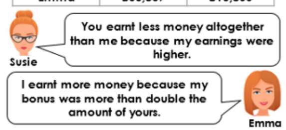
	?	4	?	3	?
+	2	?	5	?	2
	7	8	5	2	9

Complete the pyramid using addition and subtraction.

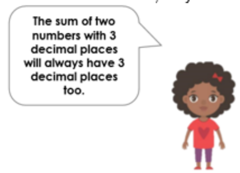


Susie and Emma are comparing their income.
Who is correct? Explain why.

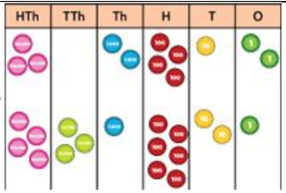
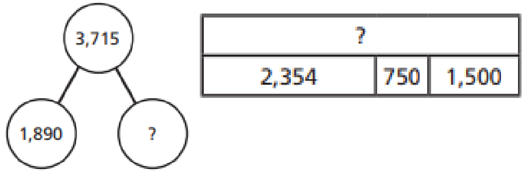
	Earnings	Bonus
Susie	£32,056	£5,099
Emma	£30,837	£10,865

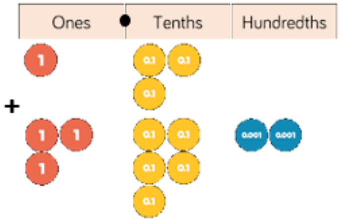
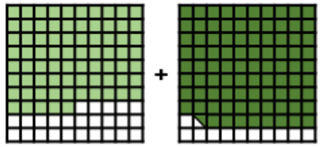
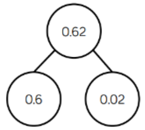
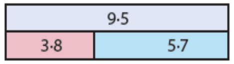


Check what Amanya has said.
Is she correct? If not, why not?



End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract

Year 6 Addition:
 	
• practise addition using the formal written methods of columnar addition
 	
• solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
 	
• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
	Column method – adding integers	
		302412 + 431621 = 734,033
		
		Use dienes or counters within a place value grid to add integers. Reinforce methods of exchanging learned in previous years.
			Use part-whole and bar models to calculate missing numbers.
			2354 + 750 + 1500 = ? (4604)
			
			3715 + 1890 = ? (5605)
				3 4 6 2 1
				+ 2 5 7 3 4
				   6 0 3 5 5
				   1 1
				
				Formal column method for exchanging and non-exchanging calculations.

	Column method – addition of decimal numbers	Reinforce place value and column method by using concrete resources and a place value chart to add decimal numbers.
		
		1.3 + 3.52 = 4.82
			Use hundred squares to help calculations.
			
			0.75 + 0.985 = 1.735
			
			
			Use pictorial methods that children are familiar with to link addition of whole numbers with the addition of decimal numbers to reinforce that the skill is the same.
				0.8 5 4
				+ 0.1 7 8
				   1.0 3 2
				   1 1
				
				Use formal written methods to add decimals, including addition of decimals and whole numbers and decimals with different numbers of decimal places.

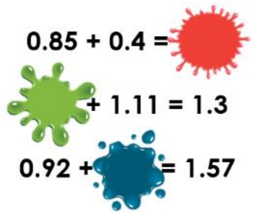
Examples of Year 6 varied fluency

	2	1		8	5
+			0	6	
	4	1	0		2

Mo wants to buy a t-shirt for £9.99, socks for £1.49 and a belt for £8.99
He has £22 in his wallet.
How could he quickly check if he has enough money?

Compare the calculations below and complete using any of the following symbols:
< > =

0.903 + 0.098		0.576 + 0.583
0.835 + 0.645		0.243 + 0.792
0.432 + 0.975		0.231 + 0.793
0.321 + 0.849		0.365 + 0.909



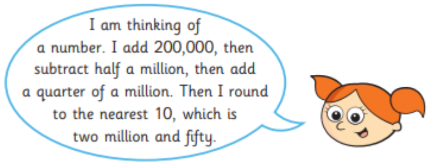
Examples of Year 6 reasoning and problem solving

Tommy has two pieces of string.

One is 160 cm long and the other is 200 cm long.

He cuts them into pieces of equal length.

What are the possible lengths the pieces of string could be?



What number could Alex have been thinking of to start with?

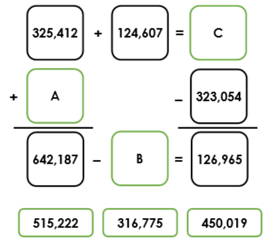
3,565 + 2,250 = 5,815

Use this calculation to decide if the following calculations are true or false.

True or False?

4,565 + 1,250 = 5,815

5,815 – 2,250 = 3,565



MENTAL STRATEGIES FOR SUBTRACTION

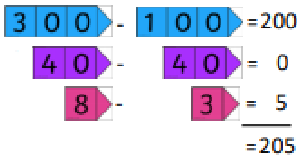
Informal jottings (for example number lines) and concrete apparatus will be used to develop an understanding of conceptual relationships

Year 3 SUBTRACTION

Using place value

Count back in hundreds e.g. knowing 372-200 as 372, 272, 172 Subtract multiples of 10, 100 and £1 e.g. 476-40 = 436 or 436-300 = 136

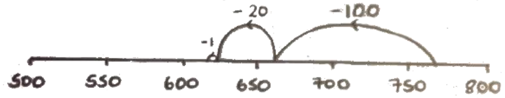
348 - 143 as 300 – 100, 40 - 40 and 8 – 3, combining the three totals. (Use arrow cards to help this method – it reinforces place value knowledge)



Counting back

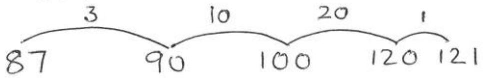
Subtract two three-digit numbers by **counting back** in h, t and o e.g. 763 - 121 as 763 - 100 – 20 – 1

Subtract near multiples of 10 and 100 e.g. 648 - 199 as (648 - 200) + 1



Counting up to find the difference

Find the difference between two numbers by **counting up** from the smallest number to the larger, e.g. 121-87



Using number facts

Number bonds to 100 e.g. 100-35, 100- 48

Year 4 SUBTRACTION

Using place value

Count back in thousands e.g. 4378-3000 as 4378, 3378, 2378, 1378

Take away multiples of 10, 100, 1000 and £1 e.g. 8392-50 or 6723-3000

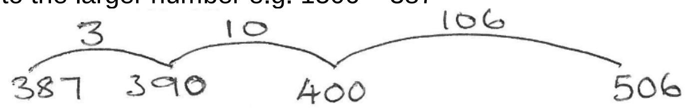
Partitioning e.g. £5.87 - £3.04 as £5 - £3 and 7p – 4p or 7493 – 2020 as 7000-2000 and 90-20

Counting back

Count back e.g. 6482 – 1301 as 6482 – 1000 then – 300 then - 1 Subtract near multiples of 10, 100 and 1000 e.g. 3522 - 1999 as (3522 - 2000) + 1

Count up to find the difference

Find a larger difference between two numbers by **counting up** from the smaller number to the larger number e.g. 1506 – 387



Using number facts

Number bonds to 100 and to the next multiple of 100 e.g. 100-76 = 24

Adjustment when adding 9 and 11 e.g. 27 - 9 as (27 - 10) +1 or 36 - 11 as (36 - 10) - 1

Year 5 SUBTRACTION

Using place value/ Taking away

Use place value to subtract decimals e.g. 4.58 – 0.08 or 6.26 – 0.2 etc.

Take away multiples of power of 10 e.g. 15,672 – 300 or 4.82-2

Partition or count back e.g. 3964 – 1051 or 5.72 – 2.01

Subtract near multiples e.g. 86,456 – 9999 or 3.58 – 1.99

Count up to find the difference

Find the difference between two numbers by **counting up** from the smaller to the larger, e.g. 2009-869

Find change using shopkeepers' addition, e.g. *buy toy for £6.89 using £10*

Use number facts

Derived facts from number bonds to 10 and 100 e.g. 2 – 0.45 using 45 + 55 = 100 Number bonds to £1, £10 and £100 e.g. £4.00 - £3.86 = 14p

Year 6 SUBTRACTION

Using place value/ Taking away

Use place value to subtract decimal numbers e.g. 7.78 – 0.08 or 16.26 – 0.2 etc.

Take away multiples of power of 10 e.g. 123,956 – 400, 686,109 – 40,000 or 7.82 – 0.5

Partition or count back e.g. 3964-1051 or 5.72 – 2.01

Subtract near multiples e.g. 360,078 – 99,998 or 12.83 – 0.99

Counting up

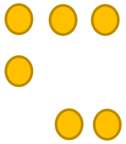
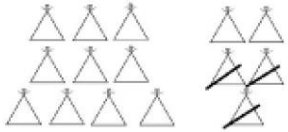
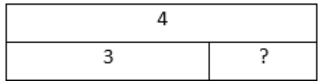
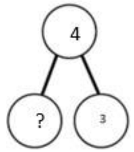
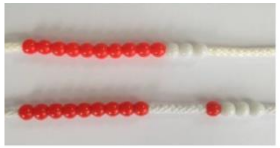
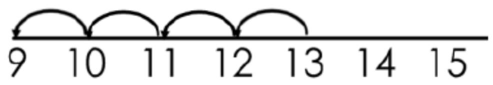
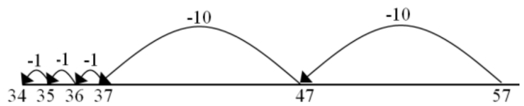
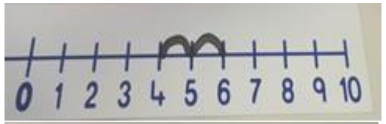
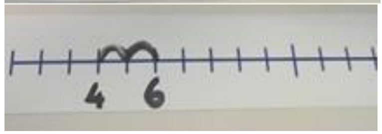
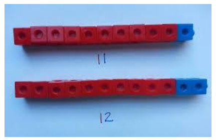
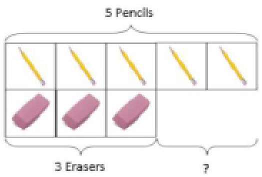
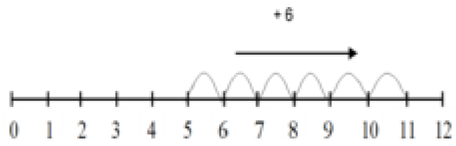
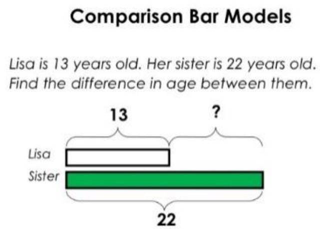
Count up to subtract numbers from multiples of 10, 100, 1000, 10,000

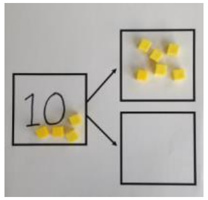
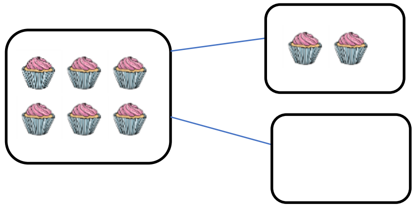
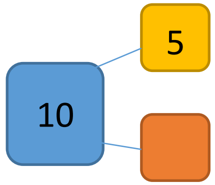
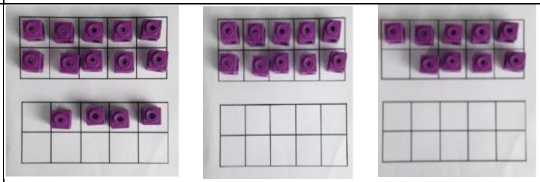
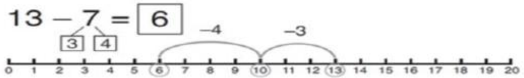
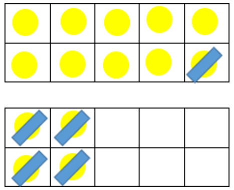
Find a difference between two decimal numbers by **counting up** from the smaller the greater number e.g. 1.2 – 0.87

Use number facts

Derive facts from number bonds to 10 and 100 e.g. 5 – 0.65

Number bonds to £1, £10 and £100 e.g. £7.00-£4.37

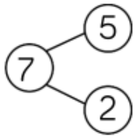
End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Key Stage 1 Subtraction: • read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • subtract one-digit and two-digit numbers to 20, including zero • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9. • solve problems with subtraction: ➢ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ➢ applying their increasing knowledge of mental and written methods • recall and use subtraction facts to 20	Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	4- 3 = 1 $\boxed{}$ = 4 – 3  
	Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4 = 9  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2-digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help or count back on a number line.  
	Find the difference	 Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference 	 Count on to find the difference. Draw bars to find the difference between 2 numbers. 	Find the difference between 8 and 6. 8 – 6, the difference is __? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 so the difference is the same- this will help when solving 10000-9987)

fluently, and derive and use related facts up to 100 • subtract numbers using concrete objects, pictorial representations, and mentally, including: ➢ a two-digit number and ones ➢ a two-digit number and tens • show that subtraction of one number from another cannot be done in any order (not commutative) • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	Part Whole Model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part whole model. 	 Move to using numbers within the part whole model.
	Make 10	 14 – 9 = Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Children to present the ten frames pictorially 	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off? Children to represent how they have solved it e.g. 14 - 5 = 9 5 5 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5 14 - 5 = 9 4 1 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9

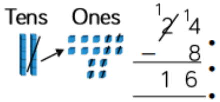
Examples of Key Stage 1 varied fluency

Can we use number bonds to subtract more efficiently?

We can partition 7 into 5 and 2 and use this to bridge the 10



Subtract 8 from 24



- Do we have enough ones to take 8 ones away?
- Exchange one ten for ten ones.
- Take away 8 ones.
- Can you write this using the column method?

34 – 13 = ____

34
30 4
−10 −3
20 1

- Partition the number 34.
- Partition 13 and subtract the ones and the tens.
- Place the partitioned number back together.

Examples of Key Stage 1 reasoning and problem solving

8 – 5 = 3
8 – 3 = 5
8 = 5 – 3
3 = 8 – 5

Rosie says,

I think that all of these facts are correct because the numbers are related

Ron disagrees.
Who is correct? Can you prove it?

Mo is counting back to solve 35 – 7

He counts

35, 34, 33, 32, 31, 30, 29

Is Mo correct?

Explain your answer.

Match the number sentences to the number bonds that make the method more efficient.

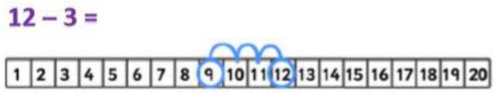
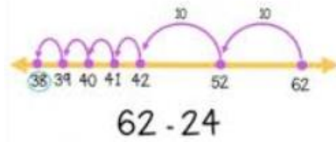
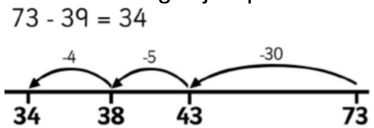
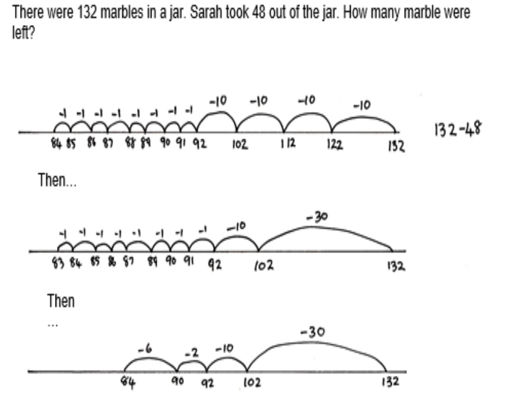
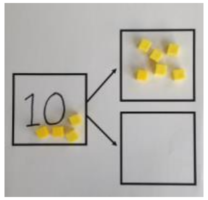
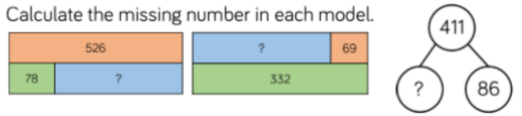
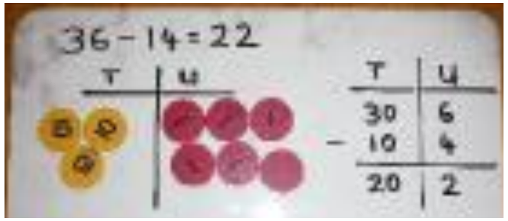
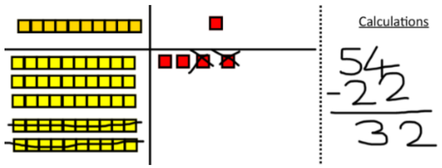
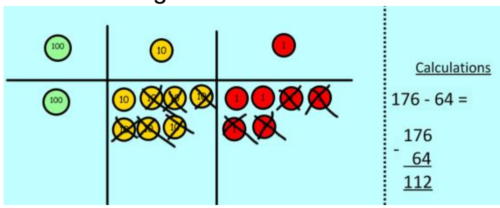
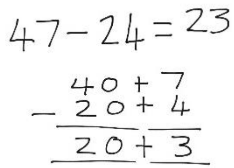
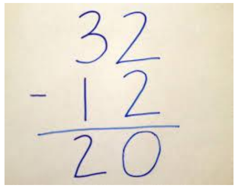
42 – 5	42 – 2 – 3
42 – 7	43 – 3 – 3
43 – 8	43 – 3 – 5
43 – 6	42 – 2 – 5

Find the missing numbers.

6 □
– 2 □
4 2

Is this the only possible solution? Explain your answer.

Make the numbers using Base 10 to help you find your answer.

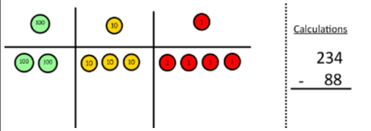
End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 3 Subtraction: • subtract numbers mentally, including: ➢ a three-digit number and ones ➢ a three-digit number and tens ➢ a three-digit number and hundreds • add and subtract numbers with up to three digits, using formal written methods of columnar subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	Subtraction on a number line	Counting back on a number line with numbers pre-drawn. (Structured number line.) 	Using a number line by counting back – unstructured number line: 1. Partition into tens and ones. 2. Subtract the tens. 3. Subtract the ones.  When children become more secure they can make larger jumps: 	Draw own number lines and decide own jumps. 
	Partitioning to subtract (use this method to reinforce place value)	 10 - 6 = 4 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part?	Use bar model and part whole pictorial representations of missing number problems for subtraction. 526 – 78 = ? (448) 332 – 69 = ? (263)  411 – 86 = ? (325)	Partitioning the numbers using the long method (expanded column method). 77 - 25 = 70 + 7 - 20 + 5 50 + 2 = 52
	Column method without exchanging HTO - O HTO - TO HTO - HTO	Use concrete equipment to make the bigger number then take the smaller number away. You can also show how you can partition numbers to subtract. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	  Begin with the expanded column method. This will lead to a clear written column subtraction.

Column method with exchanging

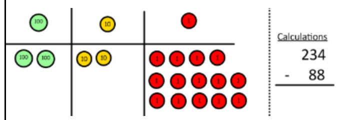
(Start with one exchange before moving onto subtractions with 2 exchanges.)

HTO - O
HTO - TO
HTO - HTO

Make the larger number with the place value counters

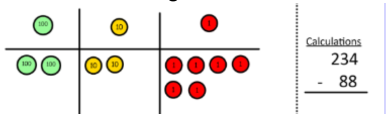


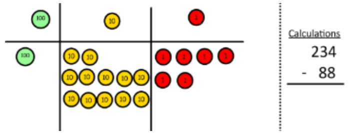
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



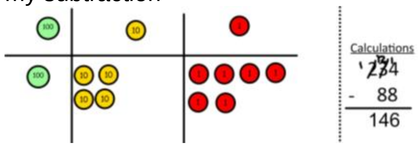
Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

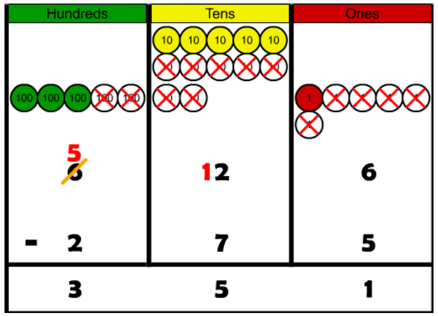




Now I can take away eight tens and complete my subtraction

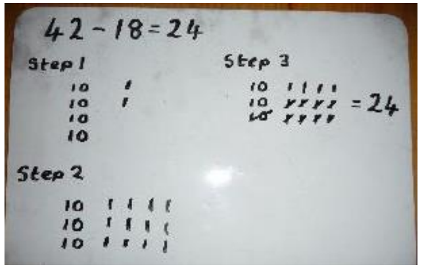


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

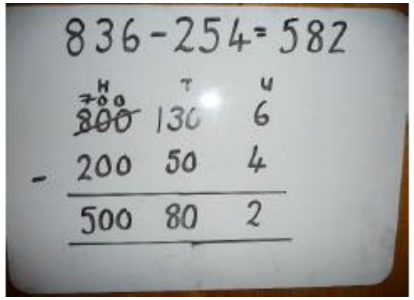


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

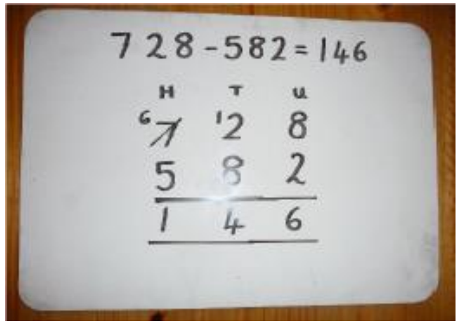
When confident, children can find their own way to record the exchange.



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange.

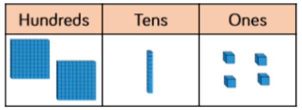


Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

Examples of Year 3 varied fluency



Use the place value grid to complete the calculations.

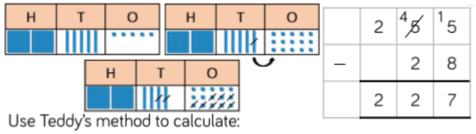
214 – 3 = ___ 214 + 3 = ___

Dora uses the part-whole model and number line to solve 132 – 4

Use this method to calculate:
132 – 8 123 – 8 123 – 5

Red team have 672 points.
Blue team have 7 fewer points than red team.
How many points do blue team have?

Teddy uses Base 10 to subtract 28 from 255



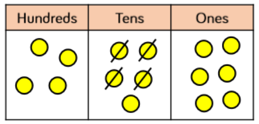
Use Teddy's method to calculate:
365 – 48 492 – 38 722 – 16

Examples of Year 3 reasoning and problem solving

If I know 700 – 500 = 200, what else do I know?

Show me using concrete and pictorial representations.

Alex thinks the chart shows 456 – 4
Do you agree?



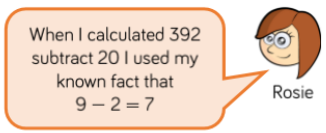
Explain why.

Explain how you would solve these calculations:

564 – ___ = 558

___ – 8 = 725

352 = 361 – ___



Explain Rosie's method.

Whitney thinks the rule for the function machine is subtract 60
Is she correct? Explain why.

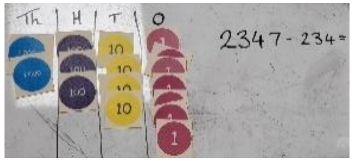
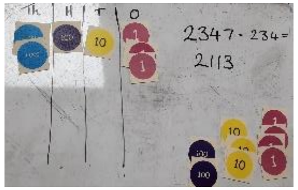
Input		Rule		Output
567	→	?	→	497

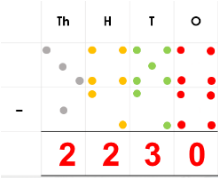
<table>
<tr><th>End of Year Expectations
(National Curriculum Statements)</th><th>Teaching and Learning Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 4 Subtraction:

• subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

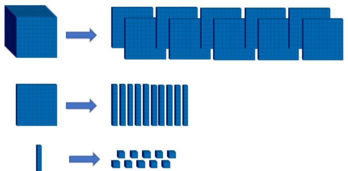
• estimate and use inverse operations to check answers to a calculation

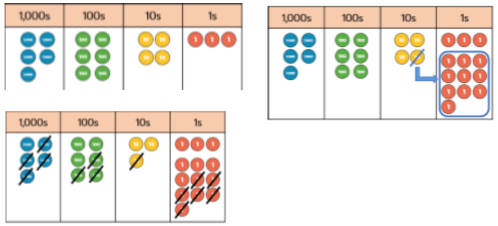
• solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</td><td>Column method without exchanging

ThHTO - O
ThHTO - TO
ThHTO - HTO
ThHTO - ThHTO</td><td>

Within a place value table, use counters or dienes to represent the minuend (the number being subtracted from) and remove the amount being subtracted.</td><td>Pictorial representation of subtraction by crossing through counters or dienes in relevant columns e.g. 3454 – 1224 = 2230


Link pictorial representations with the formal column method.</td><td>Formal column method of subtraction without exchanges. Encourage children to begin by using concrete apparatus alongside the formal written method.
<table><tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>3</td><td>4</td><td>5</td><td>4</td></tr><tr><td>–</td><td>1</td><td>2</td><td>2</td><td>4</td></tr><tr><td></td><td>2</td><td>2</td><td>3</td><td>0</td></tr></table></td></tr>
<tr><td>Column method with exchanging

(Start with one exchange before moving onto subtractions with 2 exchanges.)

ThHTO - O
ThHTO - TO
ThHTO - HTO
ThHTO - ThHTO</td><td>See Year 3 ‘Column method with exchanging’ for example on method – apply to larger numbers.

Physically exchange 1 ten for 10 ones, 1 hundred for 10 tens and 1 thousand for 10 hundreds as needed – then complete subtraction within place value grid.</td><td>Pictorial representation of exchanging and subtracting by crossing through using place value columns:

5643 – 4316 = 1327
</td><td>Formal column method of subtraction with exchanges – exchanges shown by smaller sized digits being placed into the correct column and numbers that have been exchanged from crossed out and replaced with remaining digit
<table><tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>5</td><td>6</td><td>3 ~~4~~</td><td>1 3</td></tr><tr><td>–</td><td>4</td><td>3</td><td>1</td><td>6</td></tr><tr><td></td><td>1</td><td>3</td><td>2</td><td>7</td></tr></table></td></tr>
</table>

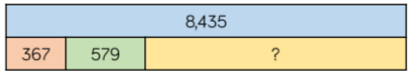
Examples of Year 4 varied fluency

Use a bar model to represent each problem.

There are 3,597 boys and girls in a school.
2,182 are boys. How many are girls?

Car A travels 7,653 miles per year.
Car B travels 5,612 miles per year.
How much further does Car A travel than Car B per year?

A shop has 8,435 magazines.
367 are sold in the morning and 579 are sold in the afternoon.
How many magazines are left?

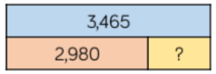


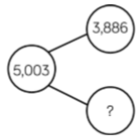
There are ____ magazines left.

Find the missing 4-digit number.

	Th	H	T	O
	?	?	?	?
+	4	6	7	8
	7	4	3	1

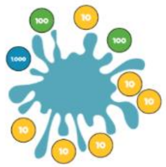
Find the missing numbers.
What methods did you use?





Examples of Year 4 reasoning and problem solving

There are counters to the value of 3,470 on the table but some have been covered by the splat.



What is the total of the counters covered?
How many different ways can you make the missing total?

1,235 people go on a school trip.

There are 1,179 children and 27 teachers.
The rest are parents.

How many parents are there?

Explain your method to a friend.

Look at each pair of calculations.
Which one out of each pair has the same difference as 2,450 – 1,830?

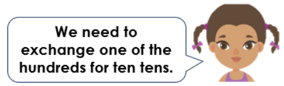
2,451 – 1,831 2,451 – 1,829

2,500 – 1,880 2,500 – 1,780

2,449 – 1,829 2,449 – 1,831

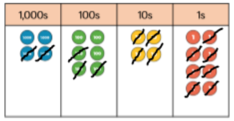
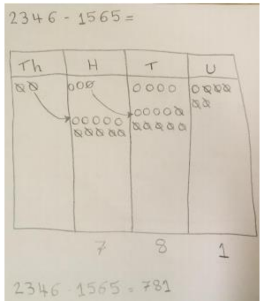
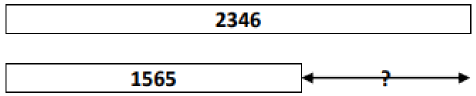
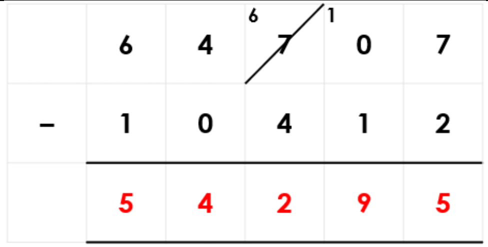
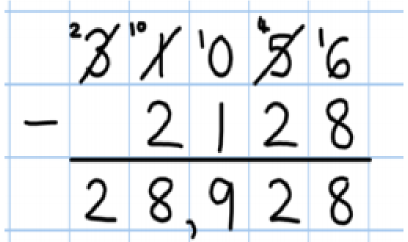
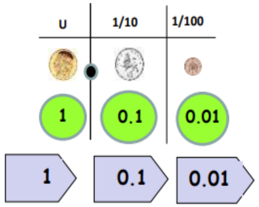
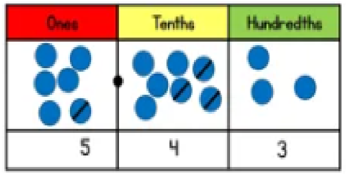
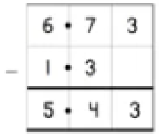
When is it useful to use difference to solve subtractions?

Class 4 are given the following calculation. Monica says,



	4	8	9	4
–	1	8	3	7

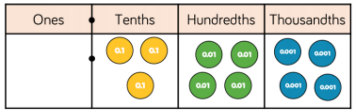
Monica is incorrect because...

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 5 Subtraction: • subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) • subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. • practise subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1	Column method – with and without exchanging TThThHTO - O TThThHTO - TO TThThHTO - HTO TThThHTO - THTO TThThHTO - TThThHTO	**See Year 3 'Column method with exchanging' for example on method – apply to larger numbers.** Use place value grids with dienes or counters to show subtraction – building on knowledge of exchanging to correctly exchange in multiple columns. To watch a video of 5 digit number subtraction with exchanging (crossing two zeros) using place value counters please click here: https://www.youtube.com/watch?v=8Tz_uc3sSCA	4,648 – 2,347   Children draw representations to help their understanding and calculations. 	 
	Column method – subtraction of decimal numbers	 Use place value grids with counters, arrow cards, dienes or money to calculate subtraction of decimals using method above.	6.73 – 1.3 = 5.43  Link pictorial methods to those used for subtraction of integers.	 Use the formal column method of subtraction for decimals. Show children that they can put 0 as a place holder so that numbers have the same number of decimal places as this may aid their confidence in using the abstract method.

Examples of Year 5 varied fluency

Represent each problem as a bar model, and solve them.

A plane is flying at 29,456 feet.
During the flight the plane descends 8,896 feet.
What height is the plane now flying at?



- What is three tenths less than the number?
- Take away 0.02, what is your number now?
- Subtract 5 thousandths. What is the final number?

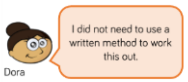
Jack has £12.54 in his wallet.
He buys a football which costs £5.82

How much money does he have left?

Examples of Year 5 reasoning and problem solving

True or False?

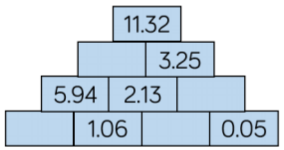
49,999 – 19,999 = 50,000 – 20,000



Can you explain why Dora's method work?

Can you think of another example where this method could be used?

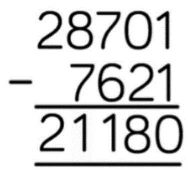
In this number pyramid, each number is calculated by adding the two numbers underneath.



Here are four calculations.
Which one is the easiest to answer?
Which one is the trickiest to answer?
Explain your choice of order.

0.45 – 0.3 =
0.45 – 0.15 =
0.45 – 0.23 =
0.45 – 0.18 =

Rosie completes this subtraction incorrectly.



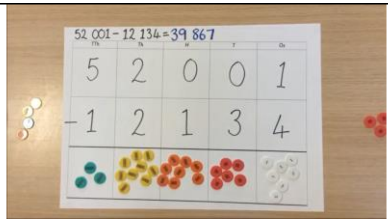
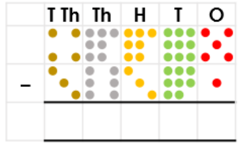
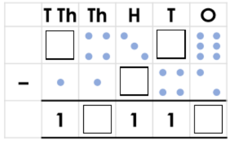
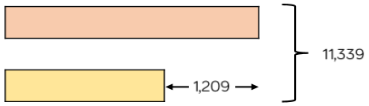
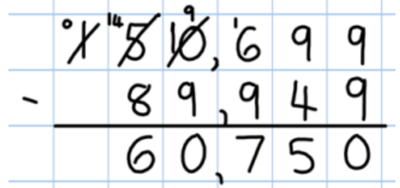
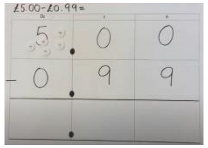
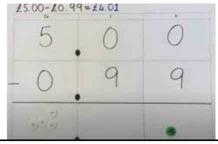
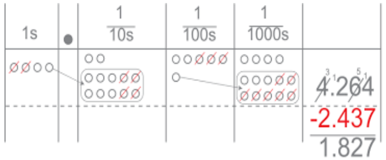
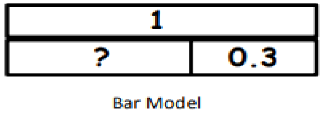
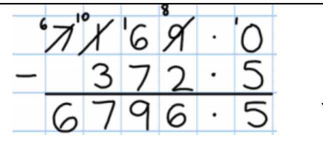
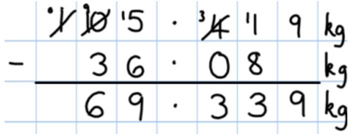
Explain the mistake to Rosie and correct it for her.

Eva makes a 5-digit number.

Mo makes a 4-digit number.

The difference between their numbers is 3,465

What could their numbers be?

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 6 Subtraction: • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. • practise calculations with decimal fraction equivalents to aid fluency	Column method – subtracting integers	 Use concrete resources to support understanding of increasingly complex subtractions, including calculations within problems.	47795 - 36381 = 11414  Use pictorial representations to solve calculations or missing number problems. 	 Use the compact column method as shown in Years 4 and 5 subtracting with increasingly large and more complex numbers.
	Column method – subtraction of decimal numbers	 Use place value charts and concrete resources e.g. counters, dienes, coins to help subtract decimal numbers in context.	 Build on pictorial methods used in previous year groups	 Build on column subtraction of decimals from Year 5 and apply skill to wider context e.g. measures.

Examples of Year 6 varied fluency

True or false?

$$\begin{array}{r} 5\ 5\ 8\ 7\ 2 \\ -\ 2\ 2\ 4\ 9\ 1 \\ \hline 3\ 3\ 3\ 8\ 1 \end{array}$$

Find the sum of and the difference between the numbers below.

782,664 **205,480**

A	384,775	120,024
eight hundred and seventy-five thousand		

Examples of Year 6 reasoning and problem solving

Jonathan chooses a number between 25,000 and 30,000. He subtracts 10,124.
His answer is 15,483.

Is Jonathan correct? Explain your answer.

The answer to a subtraction calculation using two 6-digit numbers is 130,509.

For one of the numbers, the tens digit is odd.

What could the calculation be?

Use the digit cards to complete the subtraction.

0 2 6 2 7

Jazmin chooses a number between 200,000 and 300,000.
She subtracts 123,468 and then adds 156,812. Her answer is 274,216.

Is Jazmin correct? Explain your answer.

<u>*MENTAL STRATEGIES FOR MULTIPLICATION*</u>

Informal jottings (for example number lines) and concrete apparatus will be used to develop an understanding of conceptual relationships

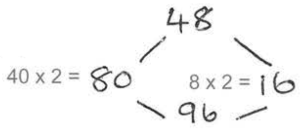
Year 3 MULTIPLICATION

Counting in steps

Count in 2s, 3s, 4s, 5s, 8s and 10s e.g. colour the multiples on a 100 square or jumps on a landmarked number line

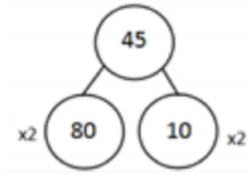
Doubling and halving

Find doubles to double 50 using partitioning
Use doubling as a strategy in multiplying by 2 e.g. 18 x 2 is double 18.



Grouping

Recognise that multiplying is commutative e.g. 8 x 4 = 4 x 8
Multiply multiples of 10 by a single-digit using knowledge of known facts e.g. 30 x 8 = 240 (3 x 8=24 then 24 x 10 = 240)
Multiply friendly two-digit numbers by single digit numbers e.g. 13 x 4



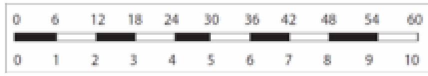
Known facts

Know 2 x, 3 x, 4 x, 5 x, 8 x and 10 x

Year 4 MULTIPLICATION

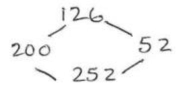
Counting in steps – sequences

Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s



Doubling and halving

Find doubles to double 100 and beyond using partitioning, using derived facts
Begin to double amounts of money e.g. £3.50 doubled is £7
Use doubling as a strategy in multiplying by 2, 4 and 8 e.g. 34 x 4 is double 34 and double again.



Grouping

Use partitioning to multiply two-digit numbers by single-digit numbers
Multiply multiples of 100 by single-digit numbers using table facts e.g. 400 x 8 = 3200 (4 x 8 = 32 then 32 x 100 = 3200)
Multiply using near multiples by rounding e.g. 24 x 19 as (24 x 20) = 24

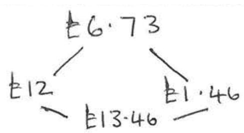
Known multiplication facts

Known facts up to 12 x 12

Year 5 MULTIPLICATION

Doubling and halving

Use double and halving strategies when multiplying by 2, 4, 8, 5 and 20
Double amounts of money using partitioning e.g. £6.73 doubled is double £6 plus double 73p



Grouping

Multiply decimals by 10, 100 and 1000 e.g. 3.4 x 100 =340
Use partitioning to multiply friendly two-digit and three-digit numbers by single digits e.g. 402 x 6 as (400 x 6) + (2 x 6)
Use partitioning to multiply decimal numbers by single digit numbers e.g. 4.5 x 3 as (4 x 3) + (4 x 0.5)
Multiply using near multiples by rounding e.g. 32 x 29 as (32 x 30) – 32

Known facts

Use times tables facts up to 12 x 12 to multiply multiples of the multiplier e.g. 6 x 4 = 24 so 60 x 4 =240 and 600 x 4 = 2400
Know square numbers and cube numbers

Year 6 MULTIPLICATION

Doubling and halving

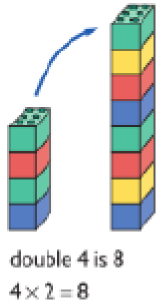
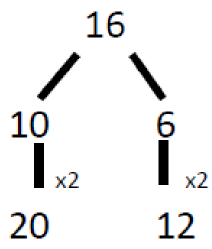
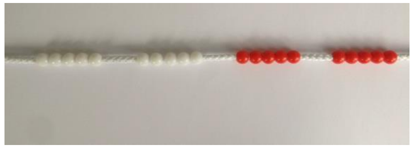
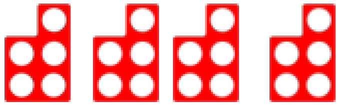
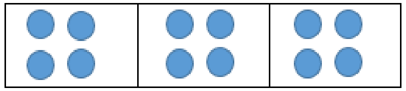
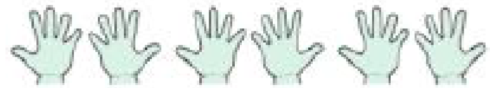
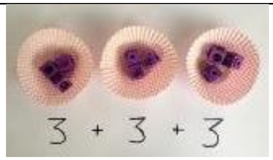
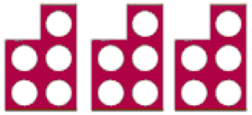
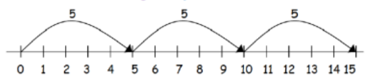
Use doubling and halving as strategies in mental multiplication.
Double decimal numbers with up to 2 places using partitioning e.g. 36.73 doubled is double 36 (72) plus double 0.73 (1.46)

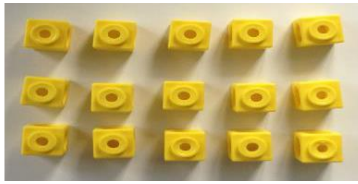
Grouping

Use partitioning as appropriate e.g. 3060 x 4 as (3000 x 4) + (60 x 4) or 8.4 x 8 as (8 x 8) + (0.4 x 8)
Use factors in mental multiplication e.g. 421 x 6 as 421 x 3 and doubled or 3.42 x 5 is half of 3.42 x 10
Multiply decimal numbers using near multiples by rounding e.g. 4.3 x 19 as (4.3 x 20) – 4.3

Known facts

Use times tables facts up to 12 x 12 in mental multiplication of large numbers or numbers with up to two-decimal places e.g. 6 x 4 = 24 so 0.06 x 4 = 0.24

End of Year Expectations (National Curriculum Statements)	**Teaching and Learning Strategy**	**Concrete**	**Pictorial**	**Abstract**
Key Stage 1 Multiplication: • solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher • recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative)	Doubling	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
	Counting in multiples – Repeated Grouping	   Count in multiples supported by concrete objects in equal groups.	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method    Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
	Repeated addition	   Use different objects to add equal groups.	  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

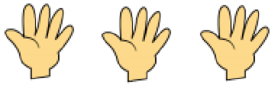
• solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts.	Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8 2 × 4 = 8 4 × 2 = 8	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Examples of Key Stage 1 varied fluency

How many flowers are there altogether?

There are _____ flowers in each bunch.
There are _____ bunches.
There are _____ flowers altogether.

How many fingers altogether?



5 + 5 + 5 =

Complete:

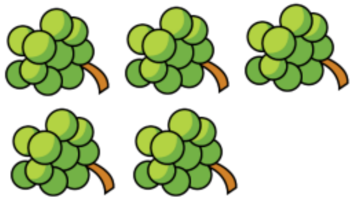
There are ____ equal groups with ____ in each group.
There are ____ 3s.
____ + ____ = 6

Complete the table.

Picture	Multiplication	Sentence
	4 × 10 = 40	4 lots of 10 is equal to 40
	35 = 7 × 5	
		6 lots of 3 is equal to 18

Examples of Key Stage 1 reasoning and problem solving

In a shop, grapes come in bunches of 10



Max wants to buy forty grapes.

Are there enough grapes?

Think of a multiplication to complete:

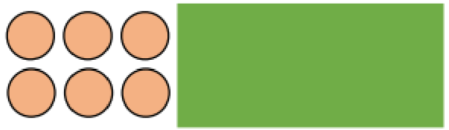
6 + 6 + 6 > ___ × ___

2 × 5
5 + 5
5 × 2

Each calculation could explain the image.

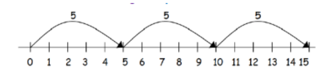
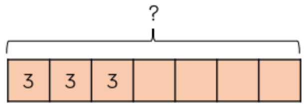
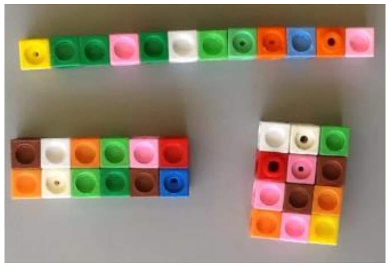
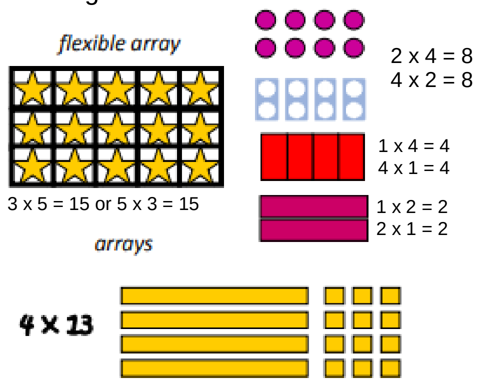
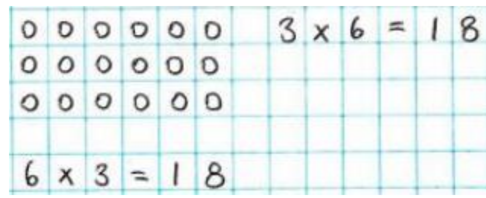
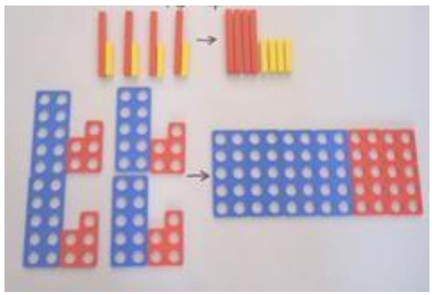
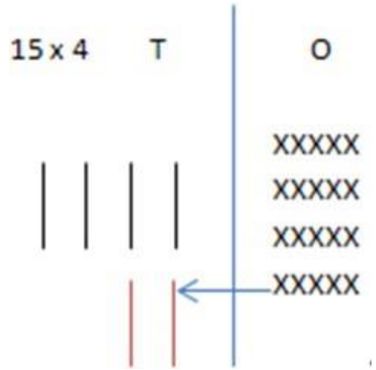
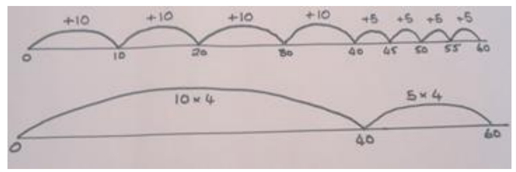
Explain why.

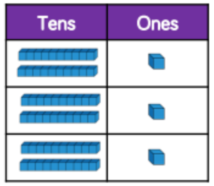
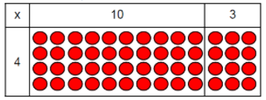
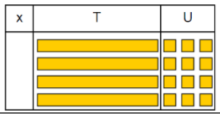
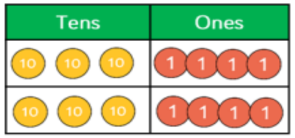
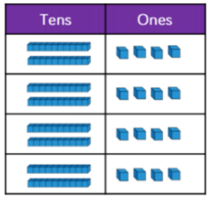
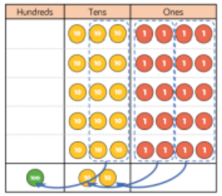
Part of this array is hidden.



The total is less than 16

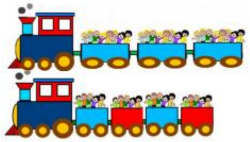
What could the array be?

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 3 Multiplication: • recall and use multiplication facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	Repeated addition	 ___ + ___ + ___ + ___ + ___ = ___ ___ × ___ = ___ Build on Key Stage 1 learning by using objects to add equal groups and make links between addition and multiplication.	Continue to use number lines (structures and unstructured) to aid multiplication through repeated addition.  5 + 5 + 5 = 15 Introduce bar model as a way of drawing repeated addition – can also be linked to an array. 	Write repeated addition sentences to describe objects and pictures and write multiplication statement to match. 4+4+4+4+4+4=24 6+6+6+6=24 6x4=24 4X6=24
	Arrays- showing commutative multiplication	Create arrays using counters/ cubes to create different multiplication sentences that have the same total e.g. 1 x 12 = 12, 12 x 1 = 12, 2 x 6 = 12, 6 x 2 =12, 3 x 4 = 12 and 4 x 3 = 12 	Draw arrays and create **commutative** multiplication sentences – building on learning from Year 2. 	Use an array to write multiplication sentences. 
	Partition to multiply	Use Numicon, base 10, Cuisenaire rods etc to physically partition the number, multiply and recombine **4 x 15** 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 	Children to be encouraged to show the steps they have taken 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used: 

		Concrete	Pictorial	Abstract
	Formal multiplication – no exchanges 2-digits by 1-digit	3 x 21 = 63  60 + 3 Use concrete resources to build groups of the largest multiplier within a place value grid. Add columns to get final answer. This layout moves children from grid method to prepare for formal written method. You can prepare children for grid method (Y4) by getting them to layout their counters like this: Show the link with arrays by using counters within a p.v. chart.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13	34 x 2 = 68  60 + 8 Children then move onto drawing representations of counters/dienes in columns in the same manner as the concrete method.	T O 3 4 × 2 6 8 Introduce children to the formal written method using numbers that do not require exchanging. Encourage children to build or draw models alongside using the written method to help develop their understanding and confidence. Children should not be doing the abstract method without pictorial or concrete resources in Year 3.
	Formal multiplication – exchanges 2-digits by 1-digit	 80 + 16 24 x 4 = 96 Use place value grid and dienes/counters to solve a multiplication that will require an exchange – link this to exchanging in addition.	 150 + 20 5 x 34 = 170 Children draw representations linked to grid method and show exchanges. (Remind children they have used this method in addition)	T O 3 4 × 5 1 7 0 1 2 Build on knowledge of formal written method, alongside concrete and pictorial methods to help children understand exchanging when answers cross into the next place value column.

Examples of Year 3 varied fluency

Describe the equal groups.



___ equal groups of ___

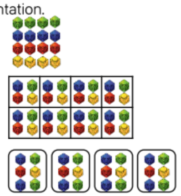
___ equal groups of ___

Match the multiplication to the representation.

4 × 4

4 × 6

8 × 4

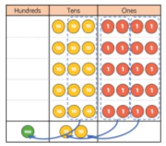


There are 3 tables with 6 children on each table.
How many children are there altogether?

___ lots of ___ = ___

___ × ___ = ___

Amir then calculates 5 × 34



	T	O
	3	4
×		5
1	7	0
1	2	

Use Amir's method to solve:
36 × 6
48 × 4

Examples of Year 3 reasoning and problem solving

Always, Sometimes, Never?

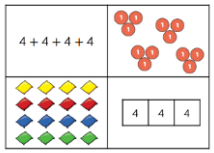
A two-digit number multiplied by a one-digit number has a two-digit product.

Dexter says,

4 × 21 = 2 × 42

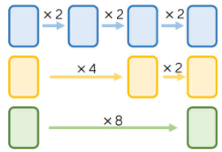
Is Dexter correct?

Which part below does not show counting in fours?



Explain why.

Start each function machine with the same number.



What do you notice about each final answer?

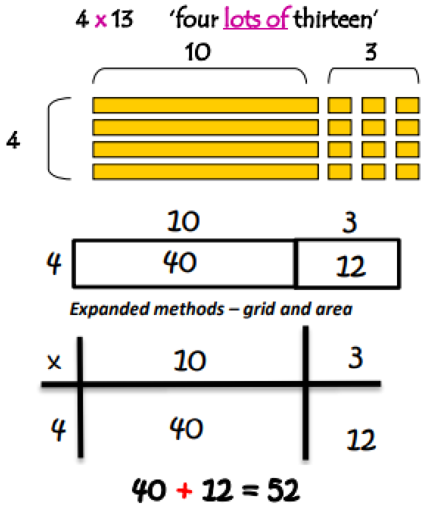
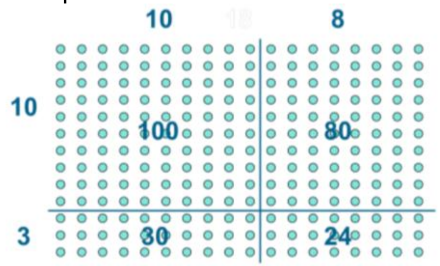
<table>
<tr>
<th>End of Year Expectations
(National Curriculum Statements)</th>
<th>Teaching and Learning Strategy</th>
<th>Concrete</th>
<th>Pictorial</th>
<th>Abstract</th>
</tr>
<tr>
<td rowspan="2">Year 4 Multiplication:

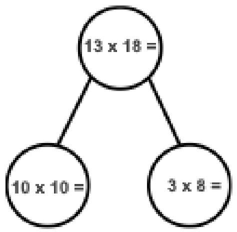
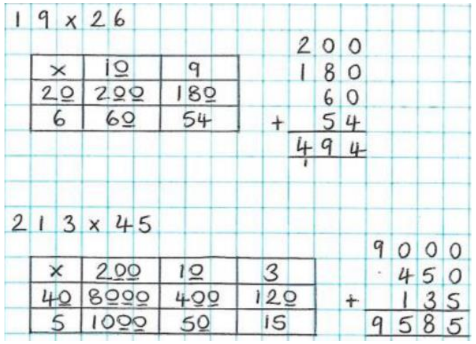
• recall multiplication facts for multiplication tables up to 12 × 12

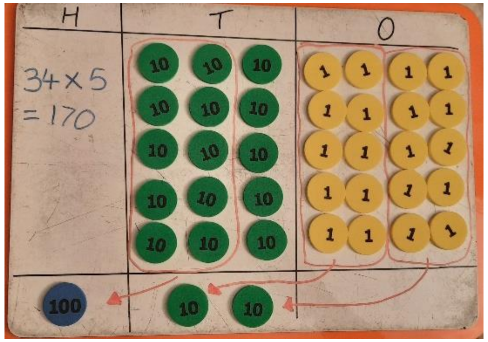
• use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers

• recognise and use factor pairs and commutativity in mental calculations

• multiply two-digit and three-digit numbers by a one-digit number using formal written layout

• solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n</td>
<td>Grid Method</td>
<td>Children use manipulatives as taught in Year 3 but begin to do written methods alongside.

</td>
<td>Use pictorial representation to help link children’s knowledge of arrays to the grid method to help consolidate their understanding of the process of multiplication. 13 x 18 = 234


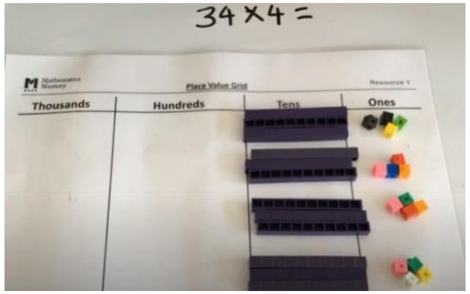
You may also want to link partitioning to the part-whole model for some children.

</td>
<td>Build on grid method from Year 3, using it to answer more complex multiplication questions.

</td>
</tr>
<tr>
<td>Long multiplication

2-digits by 1-digit
3-digits by 1-digit</td>
<td>Children can continue to be supported by place value counters at this stage of multiplication.


Children can draw round groups of ten before exchanging them for one counter in the next column.</td>
<td>

34 x 5 = 170
Children can draw representations of counters within a place value chart to help them transition to the formal column method.</td>
<td><table>
<tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td>4</td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>5</td><td></td></tr>
<tr><td></td><td></td><td>2</td><td>0</td><td>(5 × 4)</td></tr>
<tr><td>+</td><td>1</td><td>5</td><td>0</td><td>(5 × 30)</td></tr>
<tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr>
</table>Build on knowledge of formal multiplication from Year 3, begin with long multiplication alongside concrete and pictorial methods to help reinforce method and understanding.</td>
</tr>
</table>

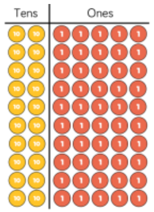
<table>
<tr>
<td>objects are connected to m objects.</td>
<td>Short multiplication

2-digits by 1-digit
3-digits by 1-digit</td>
<td>Make the largest number using concrete resources within a place value grid. Make the number of groups needed for the calculation. Add together the total.

</td>
<td>Children can draw representations of questions/problems by creating arrays within a place value grid to lead into column multiplication – this will help to establish links between prior learning and more formal methods. (This is the pictorial version of the method in the concrete column)

245 x 4 = 980

</td>
<td><table><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>2</td><td>4</td><td>5</td></tr><tr><td>×</td><td></td><td></td><td>4</td></tr><tr><td></td><td></td><td></td><td></td></tr></table>Once children are secure using long multiplication, introduce the compact version – do this alongside concrete/pictorial methods so that children can see the links between the methods and understand that the process of multiplying is the same but our method of recording looks different.
<table><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td></td><td>3</td><td>4</td></tr><tr><td>×</td><td></td><td></td><td>5</td></tr><tr><td></td><td>1</td><td>7</td><td>0</td></tr><tr><td></td><td>1</td><td>2</td><td></td></tr></table>Children will need to use their knowledge of exchanging ten ones for one ten in addition and apply this to multiplication, including exchanging multiple groups of tens.</td>
</tr>
</table>

Examples of Year 4 varied fluency



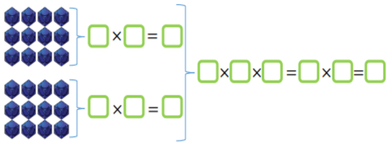
Write the calculation shown by the place value counters.

Each row has ___ tens and ___ ones.

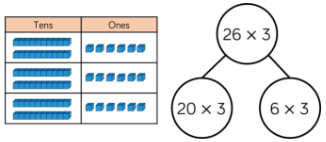
Each row has a value of ___.

There are ___ rows.

The calculation is ___ × ___ = ___.

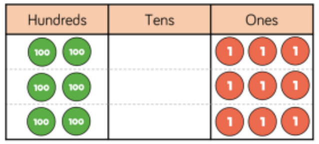


Rosie uses Base 10 and a part-whole model to calculate 26 × 3
Complete Rosie's calculations.



Use Rosie's method to work out:
36 × 3
24 × 6
45 × 4

Complete the calculation.



	H	T	O
	2	0	3
×			3

Examples of Year 4 reasoning and problem solving

Here are 6 multiplications.

43 × 5	54 × 6	38 × 6
33 × 2	19 × 7	84 × 5

Which of the multiplications would you calculate mentally?
Which of the multiplications would you use a written method for?

Here are three incorrect multiplications.

	T	O
	6	1
×		5
	3	5

	T	O
	7	4
×		7
4	9	8

	T	O
	2	6
×		4
8	2	4

Correct the multiplications.

Alex and Dexter have both completed the same multiplication.

Alex

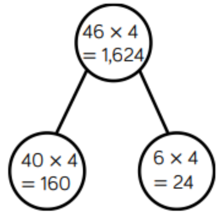
	H	T	O
	2	3	4
×			6
1	2	0	4
	2	2	

Dexter

	H	T	O
	2	3	4
×			6
1	4	0	4
	2	2	

Who has the correct answer?
What mistake has been made by one of the children?

Ron is calculating 46 multiplied by 4 using the part-whole model.



Can you explain Ron's mistake?

<table>
<tr><th>End of Year Expectations
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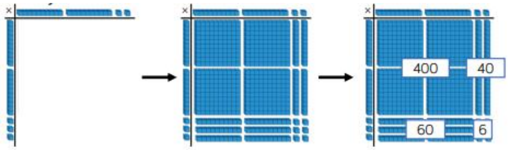
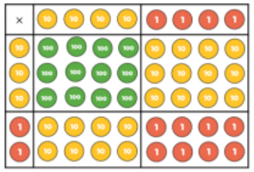
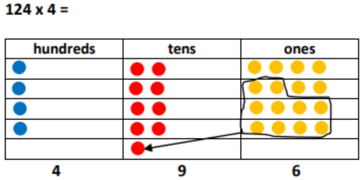
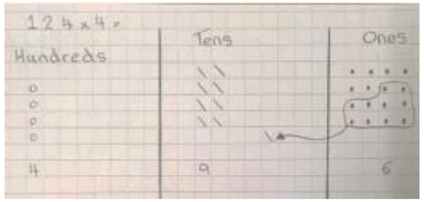
<tr>
<td rowspan="3">Year 5 Multiplication:
• identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
• multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
• multiply numbers mentally drawing upon known facts
• multiply whole numbers and those involving decimals by 10, 100 and 1000
• recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
• solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes.</td>
<td>Grid method

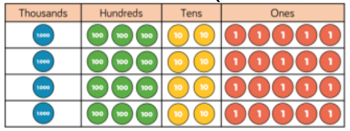
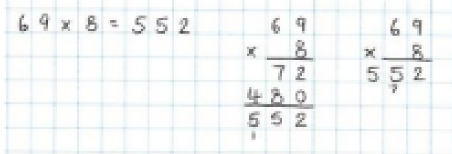
4-digits by 1-digit
2-digits by 2-digits
3-digits by 2-digits
4-digits by 2-digits</td>
<td>Use base 10 to build an area model
23 x 22 =

</td>
<td>
Build on pictorial grid method from previous years with more complex calculations.
44 x 32 = 1408</td>
<td><table><tr><td>×</td><td>40</td><td>4</td></tr><tr><td>30</td><td>1,200</td><td>120</td></tr><tr><td>2</td><td>80</td><td>8</td></tr></table>
Use written grid method to multiply 2-digit by 2-digit.

44 x 32 = 1408</td>
</tr>
<tr>
<td>Long multiplication

4-digits by 1-digit
2-digits by 2-digits
3-digits by 2-digits
4-digits by 2-digits</td>
<td>

Continue to use concrete resources in a place value grid to group and add answers to multiplication questions. Build on prior learning by using increasingly complex calculations including larger numbers and more exchanges.</td>
<td>

Children can draw representations of counters within a place value grid to help them calculate the answer and understand the process of multiplication with larger numbers.</td>
<td><table><tr><td></td><td></td><td>2</td><td>3</td><td></td></tr><tr><td>×</td><td></td><td>1</td><td>4</td><td></td></tr><tr><td></td><td></td><td>9 [1]</td><td>2</td><td>(23 × 4)</td></tr><tr><td></td><td>2</td><td>3</td><td>0</td><td>(23 × 10)</td></tr><tr><td></td><td>3</td><td>2</td><td>2</td><td></td></tr></table>
Build on Year 4 learning by using expanded column multiplication for 2-digit by 2-digit numbers.
Reinforce place value and using 0 as a place holder to ensure answers for each stage go into the right column.</td>
</tr>
<tr>
<td>Short multiplication

4-digits by 1-digit
2-digits by 2-digits
3-digits by 2-digits
4-digits by 2-digits</td>
<td>Use area models or concrete resources within a place value grid to support more complex calculations.
23
× 14
</td>
<td>Continue to build on previous pictorial methods to support more complex calculations/problems e.g. Annie earns £1,325 per week. How much would she earn in 4 weeks? (1325 x 4 = 5300)

</td>
<td>

When children are confident at calculating multiplications with larger numbers, they can move on to using the compact formal method.</td>
</tr>
</table>

Examples of Year 5 varied fluency

Calculate:

38 × 12 | 39 × 12 | 38 × 11

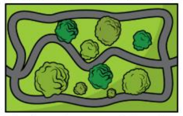
What's the same? What's different?

Use <, > or = to make the statements correct.

4,458 × 56 ◯ 4,523 × 54

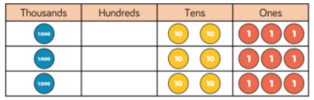
4,458 × 55 ◯ 4,523 × 54

A playground is 128 yards by 73 yards.



Calculate the area of the playground.

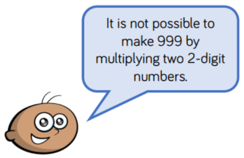
Complete the calculation.



Th	H	T	O
1	0	2	3
×			3

Examples of Year 5 reasoning and problem solving

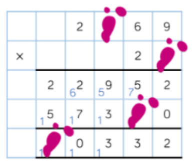
Tommy says,



Do you agree?
Explain your answer.

Pencils come in boxes of 64
A school bought 270 boxes.
Rulers come in packs of 46
A school bought 720 packs.
How many more rulers were ordered than pencils?

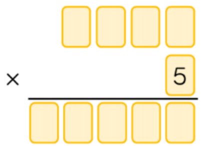
Teddy has spilt some paint on his calculation.



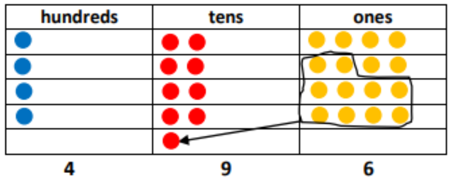
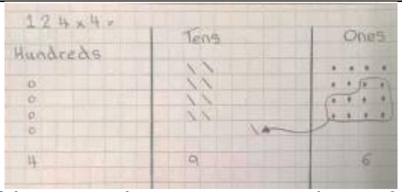
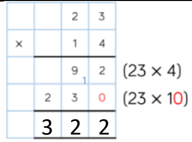
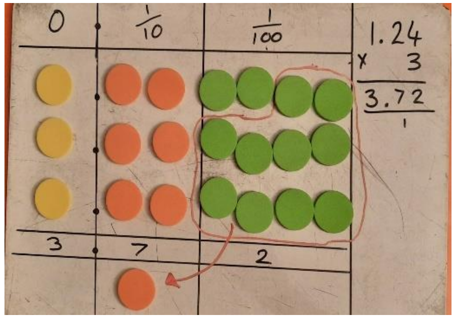
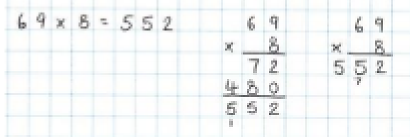
What are the missing digits?

What do you notice?

Can you work out the missing numbers using the clues?



- The 4 digits being multiplied by 5 are consecutive numbers.
- The first 2 digits of the product are the same.
- The fourth and fifth digits of the answer add to make the third.

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 6 Multiplication: • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • multiply numbers by 10, 100 and 1000 giving answers up to three decimal places	Long multiplication 4-digits by 1-digit 2-digits by 2-digits 3-digits by 2-digits 4-digits by 2-digits Decimals by integers	**124 x 4 =**  Continue to use concrete resources in a place value grid to group and add answers to multiplication questions. Build on prior learning by using increasingly complex calculations including larger numbers and more exchanges.	 Children can draw representations of counters within a place value grid to help them calculate the answer and understand the process of multiplication with larger numbers.	 Build on Year 4 and Year 5 learning by using expanded column multiplication for larger numbers. Reinforce place value and using 0 as a place holder to ensure answers for each stage go into the right column. When appropriate this method can also be used to multiply decimals by integers e.g. 3 . 2 4 x 2 6 1 9 . 4 4 6 4 . 8 0 8 4 . 2 4 1 1
	Short multiplication 4-digits by 1-digit 2-digits by 2-digits 3-digits by 2-digits 4-digits by 2-digits Decimals by integers	An example of how to multiply decimals by integers using concrete equipment can be seen here: https://www.youtube.com/watch?v=MUk1VASGz8M 	Continue to build on previous pictorial methods to support more complex calculations/problems including multiplying decimals by integers: 1.212 x 3 = 3.636 	 When children are confident at calculating multiplications with larger numbers, they can move onto using he compact formal method. When appropriate this method can also be used to multiply decimals by integers. 3 . 2 4 x 6 1 9 . 4 4 1 2

Examples of Year 6 varied fluency

Calculate.

	4	2	6	7
×			3	4

	3	0	4	6
×			7	3

5,734 × 26

A jar of sweets weighs 1.213 kg.
How much would 4 jars weigh?

Work out the missing number.

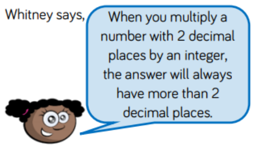
6 × 35 = ___ × 5

Jack made cookies for a bake sale.
He made 345 cookies.
The recipe says that he should have 17 raisins in each cookie.

How many raisins did he use altogether?

Examples of Year 6 reasoning and problem solving

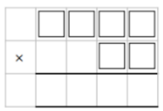
Whitney says,



Do you agree?
Explain why.

2 3 4 5 7 8

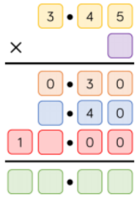
Place the digits in the boxes to make the largest product.



True or False?

- 5,463 × 18 = 18 × 5,463
- I can find the answer to 1,100 × 28 by calculating 1,100 × 30 and subtracting 2 lots of 1,100
- 702 × 9 = 701 × 10

Fill in the blanks

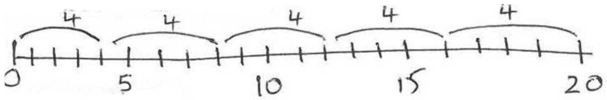


MENTAL STRATEGIES FOR DIVISION

Informal jottings (for example number lines) and concrete apparatus will be used to develop an understanding of conceptual relationships

Year 3 DIVISION

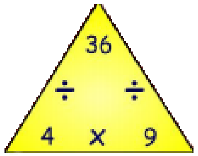
Counting in steps
Count in 2s, 3s, 4s, 5s, 8s and 10s e.g. colour the multiples on a 100 square or make jumps on a landmarked number line e.g. 20 ÷ 4 =



Doubling and halving
Find half of even numbers to 100, using partitioning
Use halving as a strategy in dividing by 2 e.g. 36 ÷ 2 is half of 36

Grouping
Recognise that dividing is not commutative e.g. 16 ÷ 2 is not equal to 2 ÷ 16
Relate division to multiplication 'with holes in' e.g. ? x 5 = 30 is the same as 30 ÷ 5 = ? therefore we can count in 5s to find the answer Divide multiples of 10 by a single-digit e.g. 240 ÷ 8 = 30 (24÷8=3)

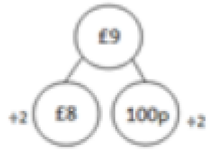
Known facts
Know half of even numbers to 40 and of 0 to 200 e.g. half of 170 is 85
Know 2 x, 3 x, 4 x, 5 x, 8 x and 10 x division facts – use triangular cards to help reinforce related facts e.g.
https://www.topmarks.co.uk/Flash.aspx?f=triangularcardsv4



Year 4 DIVISION

Counting in steps – sequences
Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s

Doubling and halving
Find halves of even numbers to 200 and beyond using partitioning, using derived facts Begin to halve amounts of money e.g. £9 halved is £4.50
Use halving as a strategy in dividing by 2, 4 and 8 e.g. 164 ÷ 4 is half of 164 and halved again.

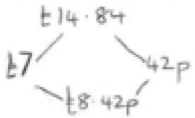


Grouping
Partition the dividend into known facts e.g. 45 ÷ 3 as 30 ÷ 3 and 15 ÷ 3
Divide multiples of 100 by single digits numbers using division facts e.g. 3200 ÷ 8 = 400

Known facts
Know times tables up to 12 x 12 and all related division facts
Use division facts to find unit and non-unit fractions of amounts

Year 5 DIVISION

Doubling and halving
Halve amounts of money using partitioning e.g. half of £14.84 is half of £14 and half of 84p
Use doubling and halving as a strategy in dividing by 2, 4, 8, 5 and 20 e.g. 115 ÷ 5 as double 115 ÷10

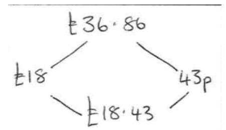


Grouping
Divide numbers by 10, 100, 1000 to obtain decimal answers with up to three places e.g. 340 ÷ 100 = 34
Use 10th, 20th, 30th multiples of the divisor to divide friendly two-digit and three-digit numbers e.g. 186 ÷ 6 as 180 ÷ 6 plus 6 ÷ 6
Find unit and non-unit fractions of large amounts e.g. 3/5 of 265 is 3 x (265 ÷ 5)

Known facts
Use division facts up to 12 x 12 to divide numbers by single-digit numbers e.g. 3600 ÷ 9 as 36 ÷ 9
Know square numbers and cube numbers

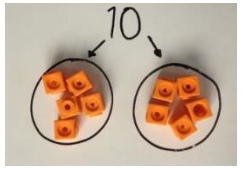
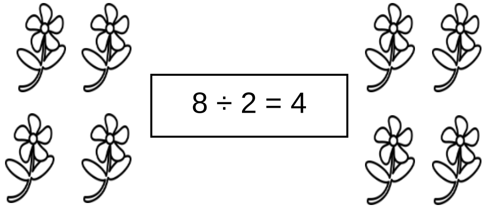
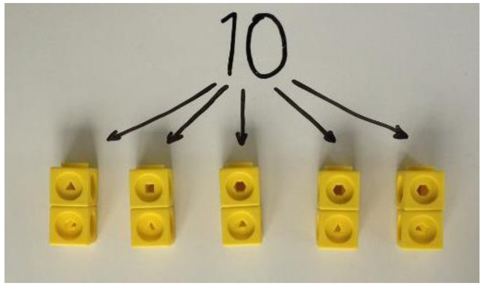
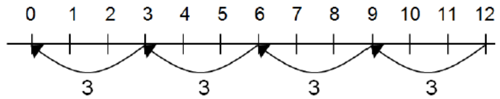
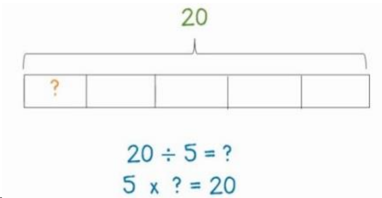
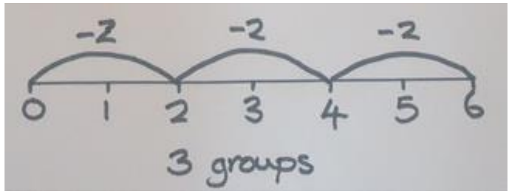
Year 6 DIVISION

Doubling and halving
Use doubling and halving as strategies in mental division. Halve decimal numbers with up to two-places using partitioning e.g. half of 38.86 is half of 38 plus half of 0.86.



Grouping
Use 10th, 20th, 30th … 100th, 200th etc. multiples of the divisor to divide larger numbers e.g. 378 ÷ 9 as 360 ÷ 9 and 18 ÷ 9.
Use tests for divisibility e.g. 135 divides by 3 as 1 + 3 + 5 = 9 and 9 is in the 3 x tables.

Known facts
Use division facts up to 12 x 12 to divide decimal numbers by single-digit numbers e.g. 1.17 ÷ 3 1/100 of 117 ÷ 3 (0.39)

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Key Stage 1 Division: • solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. • recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs • show that division of one number by another cannot be done in any order (not commutative) • solve problems involving division, using materials, arrays, mental methods, and multiplication and division facts, including problems in contexts.	Sharing objects into groups	   I have 10 cubes (or other objects), can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 	Share 9 buns between three people. 9 ÷ 3 = 3
	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? Abstract number line. 

Examples of Key Stage 1 varied fluency

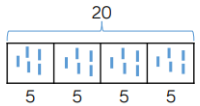
Take 20 cubes. Complete the sentences.
I can make ___ equal groups of 2
I can make ___ equal groups of 5
I can make ___ equal groups of 10

Group the socks into pairs.

□ ÷ □ = □
□ × □ = □

Complete the number sentences.

Ron draws this bar model to divide 20 into 4 equal groups.
How does his model represent this?
He writes 20 ÷ 4 = 5



What other number sentences could Ron create using his model?

Examples of Key Stage 1 reasoning and problem solving

True or false?

Dividing by 10 is the same as dividing by 5 then dividing by 2

Tommy and Jack each have the same number of sweets.

Tommy has 5 equal groups of 2
Jack has 1 equal group.
How many sweets are in Jack's group?

You have 30 counters.

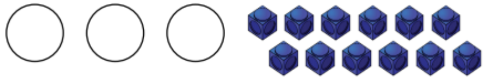
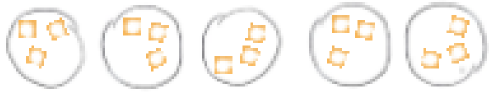
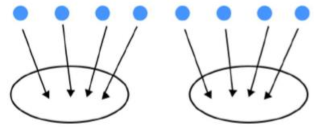
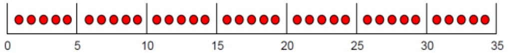
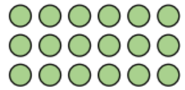
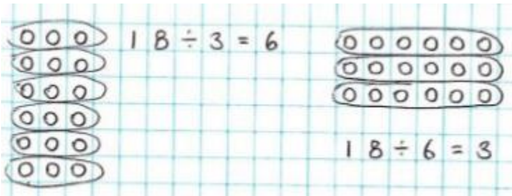
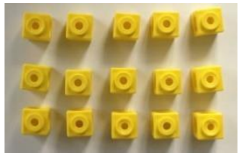
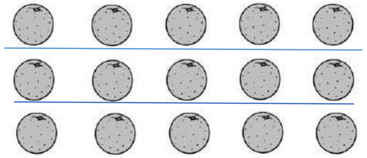
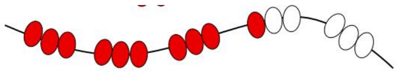
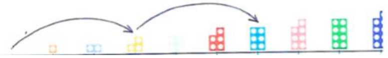
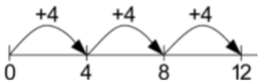
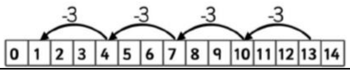
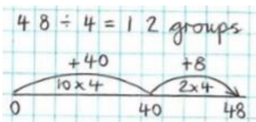
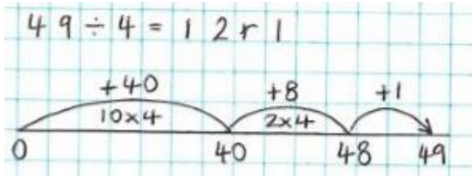
How many different ways can you put them into equal groups?

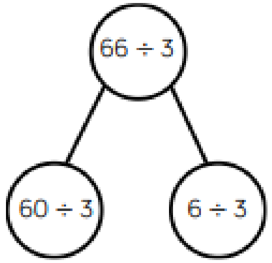
Write down all the possible ways.

I have 24p.
I divide it equally between 2 friends.
How much will they get each?

I have 24p in 2p coins.
How many 2p coins do I have?

Consider the two questions above.
What is the same and what is different?

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 3 Division: • recall and use division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	Sharing objects into groups	Use concrete resources (cubes, counters etc) to represent numbers/objects in a division problem and share into a given number of groups e.g. There are 12 pieces of fruit. They are shared equally between 3 bowls. How many pieces of fruit are in each bowl? 12 ÷ 3 = 4 	Draw circles to represent number of groups and share out by drawing dots/shapes. 15 ÷ 5 = 3 	Begin to link division number sentences to concrete and abstract methods. 8 ÷ 2 **Sharing equally** 8 sweets shared between 2 people, how many do they each get? 
	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32 	Draw representations of the question to group the total to find the answer to the division. Circle the counters in groups of 3 and complete the division.  ___ ÷ 3 = ___ 18 ÷ 3 = 6	How many 3's are in 18? 18 ÷ 3 can be modelled as: 
	Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences.  15 ÷ 3 = 5	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
	Division on a number line	Group from zero in jumps of the divisor to find 'how many groups of 3 are there in 15?'. 15 ÷ 3 = 5   	Repeated addition or subtraction using a number line with the numbers pre-drawn on. 12 ÷ 4 =  This method can also be used when looking at division with answers involving remainders. 13 ÷ 3 = 4r1 	 Children need to be able to partition the dividend in different ways. 

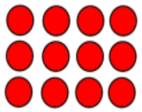
	Division by partitioning 2-digits by 1-digit	Children divide 2-digit numbers by a 1-digit number by partitioning into tens and ones and sharing into equal groups. 66 ÷ 3 = 22	Link this to previous learning of partitioning in multiplication and use of the part-whole model. 	Look for ways to partition the dividend so that known multiplication facts can be used to complete the calculation. **84 ÷ 7** 84 70 + 14 ÷ 7 10 + 2 = 12

Tens		Ones	
10	10	1	1
10	10	1	1
10	10	1	1

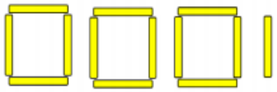
Examples of Year 3 varied fluency

Use the array to complete the number sentences.

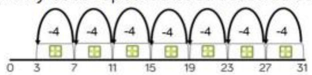
3 × 4 = ☐
4 × 3 = ☐
☐ ÷ 3 = ☐
☐ ÷ 4 = ☐



How many squares can you make with 13 lollipop sticks?
There are ___ lollipop sticks.
There are ___ groups of 4
There is ___ lollipop stick remaining.
13 ÷ 4 = ___ remainder ___



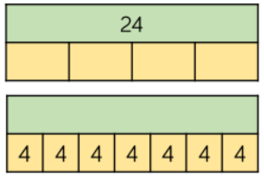
Tommy uses repeated subtraction to solve 31 ÷ 4



31 ÷ 4 = 7 r 3

Use Tommy's method to solve 38 divided by 3

Complete the bar models and the calculations.



24 ÷ 4 = ___

___ ÷ 4 = ___

Examples of Year 3 reasoning and problem solving

Dora thinks that 88 sweets can be shared equally between eight people.

Is she correct?

Compare the statements using <, > or =

48 ÷ 4 ◯ 36 ÷ 3

52 ÷ 4 ◯ 42 ÷ 3

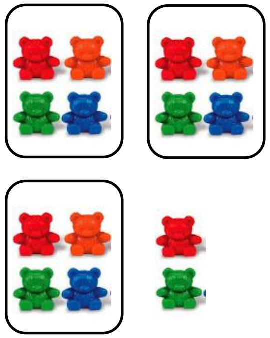
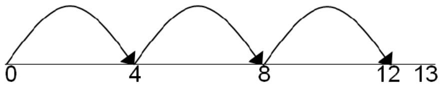
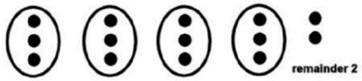
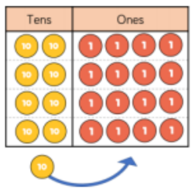
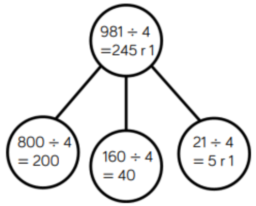
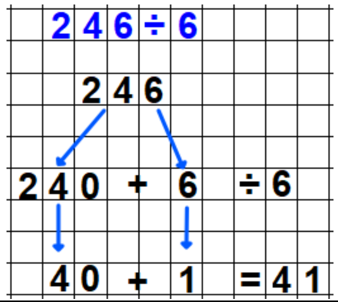
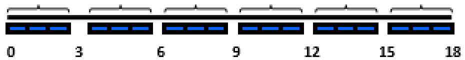
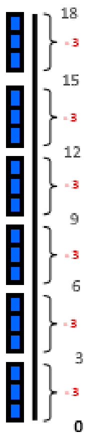
60 ÷ 3 ◯ 60 ÷ 4

Which calculation is the odd one out?
Explain your thinking.

64 ÷ 8	77 ÷ 4
49 ÷ 6	65 ÷ 3

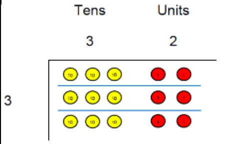
Jack has 15 stickers.

He sorts his stickers into equal groups but has some stickers remaining.
How many stickers could be in each group and how many stickers would be remaining?

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 4 Division: • recall division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to divide mentally, including dividing by 1 • solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers	Division with and without a remainder	14 ÷ 4 = 3 r2 Divide objects between groups and see how much is left over 	Build on learning from Year 3 to become more confident in finding reminders after division.Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r1  Draw dots and group them to divide an amount and clearly show a remainder.  14 ÷ 3 = 4 r2	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder
	Division by partitioning 2-digits by 1-digit 3-digits by 1-digit	Build on Year 3 division by partitioning by exchanging e.g. Rosie is calculating 96 divided by 4 using place value counters. First, she divides the tens. She has one ten remaining so she exchanges one ten for ten ones. Then, she divides the ones.  96 ÷ 4 = 24	Link this to previous learning of partitioning in multiplication and use of the part-whole model. Begin to use with larger numbers and calculations involving remainders. 	 Look for ways to partition the dividend so that known multiplication facts can be used to complete the calculation.
	Repeated subtraction – chunking 2-digits by 1-digit 3-digits by 1-digit	 Ensure children see/understand the link between grouping on a number line and vertical recording for chunking.  18 ÷ 3 = 6	Introduce the vertical method (also known as the chunking method) using small jumps that children could draw if needed. 72 ÷ 3 = 3) 72 - 30 (10 x 3) 42 - 30 (10 x 3) 12 - 6 (2 x 3) 6 - 6 (2 x 3) 10 + 10 + 2 + 2 0 = 24	This is then developed by subtracting larger ‘chunks.’ Children should continue to use concrete resources alongside this algorithm. Teach children to write ‘Fact Boxes’ of known multiplication facts to help them to take away larger ‘chunks’. 289 ÷ 8 7) 2 8 9 - 8 0 10 x 8 2 0 9 - 1 6 0 20 x 8 4 9 - 4 8 6 x 8 1 Answer = 36 r1

Short division

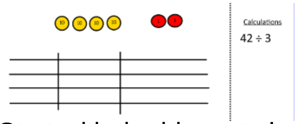
2-digits by 1-digit
3-digits by 1-digit



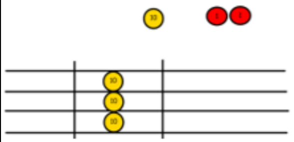
← Write quotient (answer) on top
← Use place value counters to represent the dividend.

↑ Write divisor at left hand side.

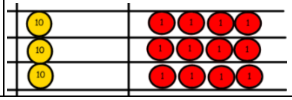
Move on to using the method with exchanges e.g. 42 ÷ 3 =



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

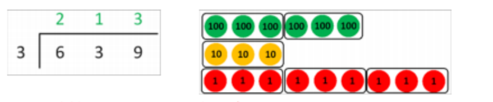


We exchange this ten for ten ones and then share the ones equally among the groups. We look how much is in 1 group so the answer is 14.



Children draw representations of place value counters alongside written methods to help reinforce understanding of division.

Without regrouping

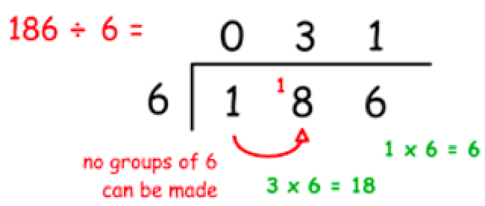


(ensure are children are grouping not sharing)

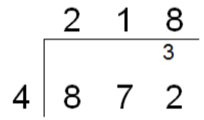
With regrouping

(children could use concrete counters or draw working out as a pictorial representation)

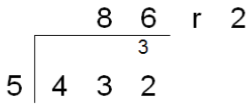
Introduce short formal division. Link to prior learning of chunking method and model how this is a compact version but the process is the same.



Begin with divisions that divide equally with no remainder.

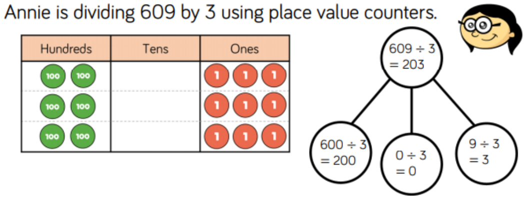


Move onto divisions with a remainder.



Examples of Year 4 varied fluency

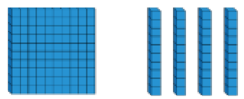
Annie is dividing 609 by 3 using place value counters.



Use Annie's method to calculate the divisions.

906 ÷ 3   884 ÷ 4   884 ÷ 8   489 ÷ 2

Use Base 10 to divide 140 by 10
Explain what you have done.



Choose the correct bar model to help you answer this question.
Annie has £4 in total. She gives away £4 at a time to her friends.
How many friends receive £4?

£4			
£1	£1	£1	£1

£4
£4

Examples of Year 4 reasoning and problem solving

Dora is calculating 72 ÷ 3
Before she starts, she says the calculation will involve an exchange.

Do you agree?
Explain why.

Use <, > or = to complete the statements.

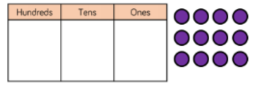
69 ÷ 3 ○ 96 ÷ 3

96 ÷ 4 ○ 96 ÷ 3

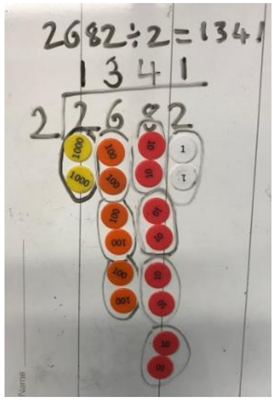
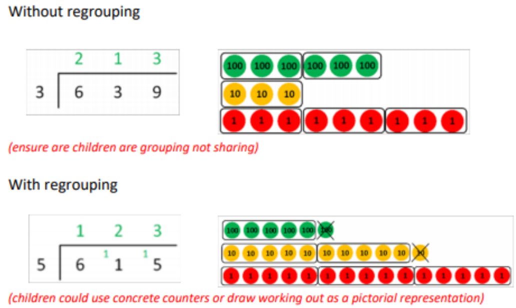
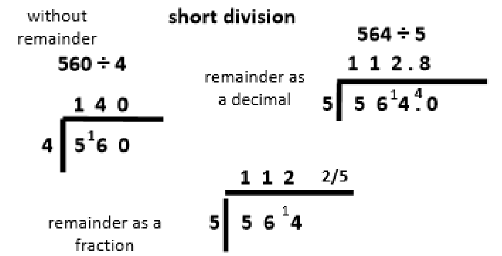
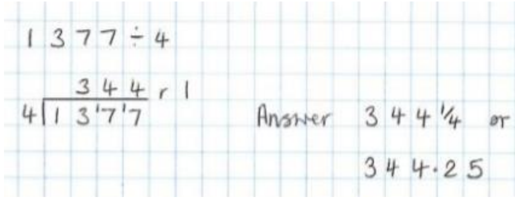
Rosie writes,
85 ÷ 3 = 28 r 1

She says 85 must be 1 away from a multiple of 3
Do you agree?

You have 12 counters and the place value grid. You must use all 12 counters to complete the following.

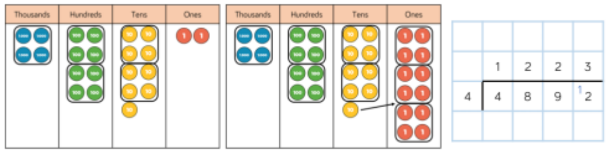


Create a 3-digit number divisible by 2
Create a 3-digit number divisible by 3
Create a 3-digit number divisible by 4
Create a 3-digit number divisible by 5
Can you find a 3-digit number divisible by 6, 7, 8 or 9?

End of Year Expectations (National Curriculum Statements)	Teaching and Learning Strategy	Concrete	Pictorial	Abstract
Year 5 Division: • divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • divide whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving division including using their knowledge of factors and multiples, squares and cubes	Short division 2-digits by 1-digit 3-digits by 1-digit 4-digits by 1-digit	Children can continue to build on concrete methods taught in previous year groups to support use of short division with larger numbers or within more complex problems. 	Build on methods taught in Year 4 and begin to use to solve more complex calculations. Children draw representations of place value counters alongside written methods to help reinforce understanding of division. 	Children begin to interpret remainders within context – they begin to understand that remainders can be written as fractions and decimals because they are equivalent.  

Examples of Year 5 varied fluency

Here is a method to calculate 4,892 divided by 4 using place value counters and short division.



Use this method to calculate:

6,610 ÷ 5   2,472 ÷ 3   9,360 ÷ 4

Use <, > or = to make the statements correct.

3,495 ÷ 5 ◯ 3,495 ÷ 3

8,064 ÷ 7 ◯ 9,198 ÷ 7

Mr Porter has saved £8,934
He shares it equally between his three grandchildren.
How much do they each receive?

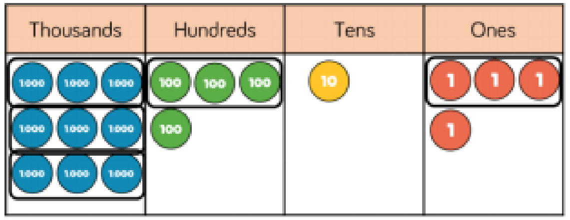
Examples of Year 5 reasoning and problem solving

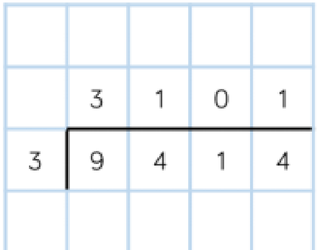
Jack is calculating 2,240 ÷ 7

He says you can't do it because 7 is larger than all of the digits in the number.

Do you agree with Jack?
Explain your answer.

Explain and correct the working.





<table>
<tr><th>End of Year Expectations
(National Curriculum Statements)</th><th>Teaching and Learning Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 6 Division:

• divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

• divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

• perform mental calculations, including with mixed operations and large numbers

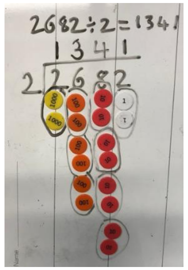
• use their knowledge of the order of operations to carry out calculations involving the four operations

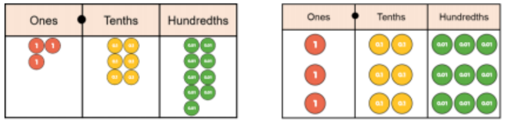
• solve problems involving addition, subtraction, multiplication and division

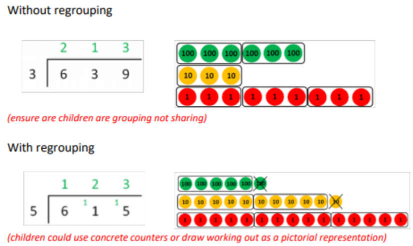
• use estimation to check answers to calculations and determine, in the context of a problem, an</td>
<td>Short division

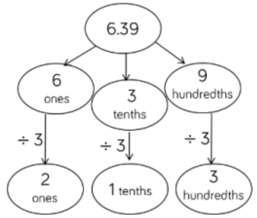
2-digits by 1-digit
3-digits by 1-digit
4-digits by 1-digit

Decimals by integers</td>
<td>Children can continue to build on concrete methods taught in previous year groups to support use of short division with larger numbers or within more complex problems.

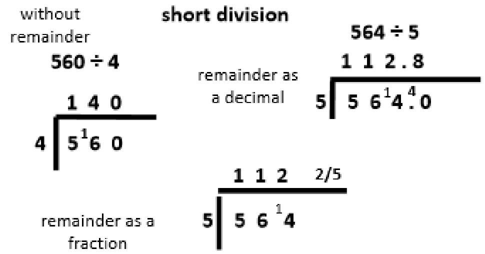


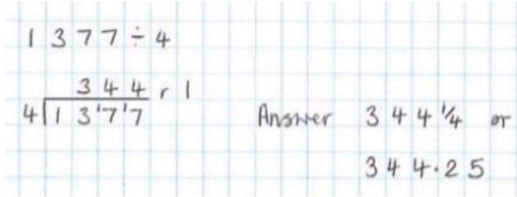


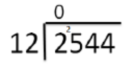
Use place value counters to divide decimals by sharing and grouping.
3.69 ÷ 3 = 1.23</td>
<td>Build on methods taught in Year 4/5 and begin to use to solve more complex calculations. Children draw representations of place value counters alongside written methods to help reinforce understanding of division.


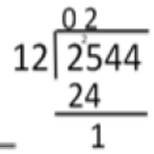


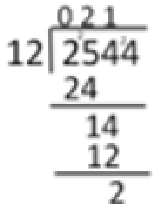
Use part-whole models to help support understanding of partitioning to divide decimals by integers.</td>
<td>Children begin to interpret remainders within context – they begin to understand that remainders can be written as fractions and decimals because they are equivalent.



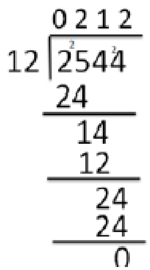

</td></tr>
<tr><td>Long division</td><td colspan="3">Link to previous concrete and pictorial methods but explain that these would not be efficient.

Step one - exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 

Step three - Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. 

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

</td></tr>
</table>

appropriate degree of accuracy.		When modelling this method make it as visual as possible to aid understanding. Use arrows to show the movement of digits and to help children correctly line up digits within place value columns. Teach children to write 'Fact Boxes' of known multiplication facts to help them to become more accurate in their division. It may also help children to write the multiplication next to each step.

0175 r 11
14 | 2461
14 (1 x 14)
106
98 (7 x 14)
81
70 (5 x 14)
11

Fact Box

2 x 14 = 28
5 x 14 = 70
10 x 14 = 140

Examples of Year 6 varied fluency

Decide whether you will use grouping or sharing and use the place value chart and counters to solve:

7.55 ÷ 5   8.16 ÷ 3   3.3 ÷ 6

Modelling clay is sold in two different shops.
Shop A sells four pots of clay for £7.68
Shop B sells three pots of clay for £5.79
Which shop has the better deal?
Explain your answer.

Year 6 has 2,356 pencil crayons for the year.

They put them in bundles, with 12 in each bundle.

How many complete bundles can be made?

Examples of Year 6 reasoning and problem solving

C is $\frac{1}{4}$ of A
B = C + 2

Use the clues to complete the division.

0 . B B
A | C . C B C 2

Each division sentence can be completed using the digits below.

1 2 3 4 5 6

[ ] . 3 ÷ [ ] = 0.26

12 . [ ] ÷ [ ] = 4.2

4 . [ ] 8 ÷ [ ] = 1.07

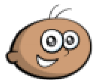
Here are two calculations.

A = 396 ÷ 11

B = 832 ÷ 13

Find the difference between A and B.

Tommy says,



To calculate 4,320 ÷ 15 I will first divide 4,320 by 5 then divide the answer by 10

Do you agree?
Explain why.